

HW Due wed

(12) email me jpg picture of yourself

(13) Suppose $A \subseteq \{1, 2, \dots, 12\}$ and $|A| \geq 9$

a) prove there is x such that $x, x+1, x+2$ are in A

b) Show by example $|A|=8$ is not good enough to prove a)

(12) done

(13) a) lets partition A s.t. $A = B_1 \cup B_2 \cup B_3 \cup B_4$
 or where $B_1 \subseteq \{1, 2, 3\}$, $B_2 \subseteq \{4, 5, 6\}$, $B_3 \subseteq \{7, 8, 9\}$, $B_4 \subseteq \{10, 11, 12\}$

By the pigeon hole principle, because we have 9 or more elements and 4 containers, then atleast 1

partition will have 3 elements in it, No matter which partition has 3 there will exist an x s.t. $x, x+1, x+2$ are in A

b) A counter example is $1, 2, 4, 5, 7, 8, 10, 11 \in A$.

⑫ Emailed from dnwilke@wisc.edu

⑬ Suppose $A \subseteq \{1, 2, 3, \dots, 12\}$ & $|A| \geq 9$

(a) Prove that there must be an integer x such that $x, x+1, x+2$ are all in A .

(b) Give an example of an A with $|A|=8$ for which there is no such x .

[proof] (a) By setting $|A|=9$, there are 3 numbers left over in the ordered set $\{1, 2, \dots, 12\}$. These numbers act as partitions of A & create 4 disjoint subsets, call them B_1, B_2, B_3, B_4 , of A .

~~or~~
Claim: One B_i has size of at least 3 (i.e., there are at least 3 sequential numbers in that B_i).

Since we know $|B_1| + |B_2| + |B_3| + |B_4| = 9$, it is impossible for one of the B_i 's to not have size of at least 3. If $|B_1| = |B_2| = |B_3| = |B_4| = 2$, then $|A| = 8$. However, we chose $|A| = 9$. Therefore one of the B_i 's has size of at least 3 & $x, x+1, x+2 \in A$. ■

~~or~~ (b) For $|A|=8$, choose $1, 2, 4, 5, 7, 8, 10, 11$. Then no $x, x+1, x+2 \in A$.