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Math 475

HW Due: Friday 2/5/10

(9) Suppose $|A| = 5$ — 9 OR

a) How many functions are there $f: A \to \{0,1,2,\cdots\}$ — 10 OR
s.t. $\sum_{a \in A} F(a) = 10$

b) How many functions are there $f: A \to \{1,2,3,\cdots\}$
s.t. $\sum_{a \in A} F(a) = 10$

(10) prove
$\binom{n}{k+1} = \binom{n-1}{k} + \binom{n-2}{k} + \cdots \binom{k+1}{k} + \binom{k}{k}$ $\quad k \le n-1$

9. a) $C(14,4) = \boxed{1001}$ $\quad$ 00$|$0000$|$000$|$0$|$00

b) $C(9,4) = \boxed{126}$

10. let S be a set of $k+1$ elements
let $A = \{S \subseteq \{1,2,\cdots,n\} \mid |S| = k+1\}$
$|A| = \binom{n}{k+1}$
choose $B_1 = \{S \in A \mid n \in S\}$
we have $n-1$ elements left and we have choosen 1
So we must choose k more.
Therefore $|B_1| = \binom{n-1}{k}$
Then choose $B_2 = \{S \in A \mid n \notin S \text{ and } n-1 \in S\}$
this leaves $n-2$ elements left and we have choosen 1
So we must choose k more. Therefore $|B_2| = \binom{n-2}{k}$
This contine until $B_{n-k} = \{S \in A \mid n, n-1, n-2, \cdots, n-k \notin S\}$
therefore $|B_{n-k}| = \binom{k}{k}$. This will complete the partitions.
Therefore $|A| = |B_1| + |B_2| + \cdots + |B_{n-k}|$ or
$\binom{n}{k+1} = \binom{n-1}{k} + \binom{n-2}{k} + \cdots + \binom{k}{k}$.