

Do not hand in this sheet. Show all work.

1. A student is to answer 7 out of 10 questions in an examination.
 - (a) How many choices has she?
 - (b) How many if she must answer at least 3 of the first 5 questions?
2. Urn A contains 3 red and 3 black balls, whereas urn B contains 4 red and 6 black balls. If a ball is randomly selected from each urn, what is the probability that the balls will be the same color?
3. You ask your neighbor to water a sickly plant while you are on vacation. Without water it will die with probability .8; with water it will die with probability .15. You are 90% certain that your neighbor will remember to water the plant.
 - (a) What is the probability that the plant will be alive when you return?
 - (b) If it is dead, what is the probability your neighbor forgot to water it?
4. A coin that when flipped comes up heads with probability p is flipped until either heads or tails has occurred twice. Find the expected number of flips.

5. The random variable X has probability density function:

$$f(x) = \begin{cases} ax + bx^2 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

If $E(X) = .6$, find (a) $P(X < \frac{1}{2})$ (b) $Var(X)$

6. The joint density function of X and Y is

$$f(x, y) = \begin{cases} xy & \text{if } 0 < x < 1, 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find the density function of X .
- (b) Find $E[Y]$
- (c) Find $P(X + Y < 1)$
- (d) Are X and Y independent?

7. Suppose that the number of automobiles sold weekly at a certain dealership is a random variable with expected value 16 and variance 9.

(a) Give a lower bound to the probability that next week's sales are between 10 and 22 inclusively.

(b) Give an upper bound to the probability that next week's sales exceed 18.

8. The servicing of a machine requires two separate steps, with the time needed for the first step being an exponential random variable with mean .2 hour and the time for the second step being an independent exponential random variable with mean .3 hour. If a repairperson has 20 machines to service, approximate the probability that all the work can be completed in 8 hours.

Answers

1. Self-test 1-4, (a) 120 (b) 110
2. Self-test 2-6, $1/2$
3. Self-test 3-8, (a) .785 (b) .372093
4. Self-test 4-3, $2 + 2p - 2p^2$
5. Self-test 5-4, (a) .35 (b) .06
6. Self-test 6-6, (a) $f(x) = 2x$ if $0 < x < 1$, $f(x) = 0$ otherwise (b) $4/3$
(c) $1/24$ (d) yes
7. Self-test 8-2, (a) $3/4$ (b) $1/2$
8. Self-test 8-7, .1074