

Do not hand in this sheet. Show all work.

1. A point is chosen at random on a line segment of length L and the line segment is then cut at that point into two subsegments. Find the probability that the ratio of the shorter to the longer subsegment is less than $\frac{1}{3}$.
2. In the following use the normal approximation to the binomial distribution (DeMoivre-Laplace). You may express your answer in terms of Φ . One thousand rolls of a fair die will be made. If the number 2 appears exactly 500 times, compute the approximate probability that the number 5 appears between 90 and 110 times.
3. If X is a random variable with the exponential distribution with parameter $\lambda = 1$. Suppose $Y = e^X$. Find the probability that $2 < Y < 3$.
4. Suppose two balls are randomly selected without replacement from an urn containing 2 white and 3 red balls. Let W be the number of white balls selected and R be the number of red balls selected. Give the joint probability mass function of W and R .
5. Three points X, Y, Z are randomly selected (uniformly and independently) in the unit interval $[0, 1]$. What is the probability that $X < YZ$?
6. An ordinary deck of playing cards is shuffled and then seven cards are dealt out one at a time. Let A be the number of aces dealt out. Find the expected value of A and the variance of A .
7. You deal out a well-shuffled pack of cards until you have seen five spades. How many cards can you expect to deal out?
8. A fair die is rolled until we roll a six. Let S be the sum of the rolls. For example, if we roll 4, 1, 4, 2, 6, then $S = 4 + 1 + 4 + 2 + 6 = 17$. Find the expected value of S .

Answers

1. $\frac{1}{2}$
2. $\Phi(\frac{10}{\sqrt{8}}) - \Phi(-\frac{10}{\sqrt{8}}) \approx .7$
3. $\frac{1}{6}$

4.

$$P(W = 2, R = 0) = \frac{1}{10}$$

$$P(W = 1, R = 1) = \frac{6}{10}$$

$$P(W = 0, R = 2) = \frac{3}{10}$$

all other combinations have probability 0.

5. $\frac{1}{4}$

$$6. E(A) = \frac{7}{13} \approx .5385$$

$$var(A) = 7(\frac{1}{13})(\frac{12}{13}) + 42(\frac{1}{13}\frac{3}{51} - \frac{1}{13}\frac{1}{13}) \approx .4386$$

$$7. 5 + 39(\frac{5}{14}) \approx 18.93$$

$$8. 21$$