

Do not hand in this sheet. Show all work.

1. A person has 9 friends of whom 5 will be invited to a party.
 - (a) How many choices are there if 2 of the friends are feuding and will not attend together?
 - (b) How many choices are there if 2 will only attend together?

2. 24 new teaching positions are to be divided among 6 schools.
 - (a) How many divisions are possible? (For example, giving 4 positions to each school counts as one division. Another division is to give all 24 positions to the first school.)
 - (b) How many division are there so that each school gets at least two new teachers?

3. A bridge hand consists of 13 cards. A Yarborough is a hand which does not contain a card which is a ten or higher. Ten or higher means Ten, Jack, Queen, King, or Ace of any suit. Many friendly bridge games follow the rule that if one of the players gets a Yarborough, then all the hands are thrown in and redealt.

What is the probability that a bridge hand is a not a Yarborough?

4. A bridge hand is void in clubs if it does not contain any clubs. There are four suits: clubs, diamonds, hearts, and spades; and each has 13 cards.

What is the probability that a bridge hand is void in at least one suit?

5. Suppose A and B are two events such that $P(A) = .2$, $P(B) = .7$ and $P(A|B) = .05$

What is the probability that

 - (a) both A and B occur
 - (b) either A or B occurs
 - (c) A occurs but B does not
 - (d) $P(B|A)$.

6. Suppose that a cancer diagnostic test is 95% accurate for those that have the disease, but only 90% accurate for those that do not. Suppose that 1% of the population have the disease. What is the probability that a person who tests positive (for cancer) has the disease?
7. All the workers at a certain company drive to work and park in the company's lot. All the cars contain either one, two, or three workers. About half the workers drive solo, i.e., their car only contains them. The expected value of the number of workers in each car is 1.6. What is the probability that a random car has three workers in it?
8. Suppose that the average number of cars abandoned weekly on a certain highway is 4. Use a Poisson distribution to approximate the probability that there will be
- (a) no abandoned cars in the next week;
 - (b) at least 2 abandoned cars over the next 3 weeks.

Answers

$$1. \text{ (a) } \binom{7}{5} + 2 \binom{7}{4} = 91 \text{ (b) } \binom{7}{5} + \binom{7}{3} = 56$$

$$2. \text{ (a) } \binom{29}{5} = 118755 \text{ (b) } \binom{17}{5} = 6188$$

$$3. 1 - \frac{\binom{32}{13}}{\binom{52}{13}}$$

$$4. 4 \frac{\binom{39}{13}}{\binom{52}{13}} - 6 \frac{\binom{26}{13}}{\binom{52}{13}} + 4 \frac{1}{\binom{52}{13}} = .0519$$

$$5. \text{ (a) } .035 \text{ (b) } .865 \text{ (a) } .165 \text{ (a) } .175$$

$$6. .0876$$

$$7. .1$$

$$8. \text{ (a) } 1.8315639 \text{ (b) } .99992013$$