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M340 Final Exam
May 13, 1993

There is a total of 150 points. Show all work.

1. (15 points) What is the determinant of $(AB)^T$?

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 6 & -1 & 0 & 0 \\ 7 & 0 & 2 & 0 \\ 2 & 1 & 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 & 1 & 4 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

2. (15 points) Let A be an $m \times n$ matrix and $X = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ an $n \times 1$ column vector. Prove that

$$AX = x_1 \text{col}_1(A) + \dots + x_n \text{col}_n(A).$$

3. (15 points)

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & -1 \end{bmatrix}$$

Find an invertible matrix P and diagonal matrix D with $A = PDP^{-1}$.

4. (15 points) Use Cramer's rule to solve:

$$x - y = 2$$

$$2x + y = 3$$

5. (15 points) Suppose A is an invertible matrix and λ is an eigenvalue of A .

- (a) Prove that $\lambda \neq 0$.
- (b) Prove that $\frac{1}{\lambda}$ is an eigenvalue of A^{-1} .

6. (15 points) Suppose $L : V \rightarrow W$ is a linear transformation, the nullspace of L has only zero vector in it ($\text{null}(L) = \{\mathbf{0}\}$), and $v_1, \dots, v_n$ are linearly independent vectors in V . Prove that $L(v_1), \dots, L(v_n)$ are linearly independent.

7. (20 points)

$$A = \begin{bmatrix} 0 & -1 & 1 & 0 & -1 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 1 & -1 & 1 & -2 \end{bmatrix}$$

- (a) Find a basis for the range space of A .
- (b) Find a basis for the null space of A .

8. (15 points) Suppose A is a 13×12 matrix, and B is a 12×13 matrix, and $C = AB$. Prove that C is not invertible.

9. (25 points) Let $\mathbf{R}^{4 \times 1}$ have the inner product defined by

$$\left\langle \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \right\rangle = x_1 y_1 + 2x_2 y_2 + 2x_3 y_3 + x_4 y_4$$

and use this inner product in the following problem. Let

$$v_1 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \end{bmatrix} \quad v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Find a vector $w \in \text{span}\{v_1, v_2, v_3\}$ other than the zero vector such that w is orthogonal to both v_1 and v_2 .