

**There is a total of 200 points. You can pick up your graded exam in VanVleck 403. Your grade will be posted on the door.**

1. (10 points) Find the inverse of  $A$  or show that the inverse does not exist.

$$A = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

2. (20 points) Suppose that  $M$  and  $N$  are invertible matrices such that  $MAN = B$ .

a. If  $H$  is a basis matrix for  $\mathcal{C}(B)$ , then what matrix is a basis matrix for  $\mathcal{C}(A)$ ?

b. If  $K$  is a basis matrix for  $\mathcal{N}(A)$ , then what matrix is a basis matrix for  $\mathcal{N}(B)$ ?

c. If  $R$  is a right inverse of  $A$ , then what matrix is a right inverse of  $B$ ?

d. If  $L$  is a left inverse of  $B$ , then what matrix is a left inverse of  $A$ ?

3. (20 points) Find a diagonal matrix  $D$  and invertible matrix  $P$  such that  $A = PDP^{-1}$  where

$$A = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$$

4. (10 points)  $\left\langle \begin{bmatrix} 1+i \\ 1 \end{bmatrix}, \begin{bmatrix} 1+i \\ 3 \end{bmatrix} \right\rangle =$

5. (10 points)  $|2 + 3i| =$

6. (10 points)  $\left\| \begin{bmatrix} i \\ 1 \end{bmatrix} \right\| =$

7. (15 points) Let  $T : V \rightarrow W$  be a mapping.
- Define  $T$  is one-to-one.
  - Prove that if  $T$  is one-to-one, then for every  $X, Y \subseteq V$

$$T(X) \cap T(Y) \subseteq T(X \cap Y).$$

8. (15 points) Prove: For all  $U, V \in \mathbb{R}^{n \times 1}$

$$|\langle U, V \rangle| \leq \|U\| \|V\|$$

9. (15 points) Prove that if  $A \in \mathbb{C}^{n \times n}$  is Hermetian (ie.  $\overline{A}^T = A$ ), then the eigenvalues of  $A$  are real.

True or false, enter your answers on the answer sheet.

- If a real matrix is both unitary and Hermetian, then it must be the identity matrix.
- If  $U$  and  $V$  are  $n \times n$  uppertriangular matrices then so is  $UV$ .
- If  $D$  is a diagonal  $n \times n$  matrix, then so is  $cD^{n+1}$  where  $c$  is a scalar.
- If  $A, B, C$  and  $D$  are  $n \times n$  matrices, then  $(AB)(CD) = A(BC)D$ .
- If  $A$  and  $B$  are  $n \times n$  matrices and  $c$  is a scalar, then  $A(cB) = B(cA)$ .
- If  $A, B$ , and  $C$  are  $n \times n$  matrices and  $A + C = C + B$ , then  $A = B$ .
- If  $A$  is an  $n \times n$  invertible matrix and  $B$  is  $n \times 1$ , then the matrix equation  $AX = B$  has a unique solution.
- If  $U$  and  $V$  are unitary  $n \times n$  matrices, then  $UV$  is a unitary matrix.
- An  $n \times n$  matrix  $U$  is unitary iff for every  $X$  which is  $n \times 1$  we have  $\|UX\| = \|X\|$ .
- If  $A$  and  $B$  are Hermetian  $n \times n$  matrices, then  $A + B$  is Hermetian.
- If  $A, B$ , and  $C$  are  $n \times n$  matrices and  $AC = BC$ , then either  $A = B$  or  $C$  is the zero matrix.
- It is possible to find  $5 \times 1$  vectors  $U$  and  $V$  such that  $\|U\|=2$ ,  $\|V\| = 3$  and  $\|U + V\| = 4$ .

22. It is possible to find  $17 \times 1$  vectors  $U$  and  $V$  such that  $\|U\|=4$ ,  $\|V\| = 2$  and  $\|U + V\| = 7$ .
23. It is possible to find  $3 \times 1$  vectors  $U$  and  $V$  such that  $\langle U, V \rangle = 5$  and  $\|U\| = \|V\| = 2$ .
24. For any  $n$  there exists an  $n \times n$  elementary matrix  $E$  such that  $EA = A^T$  for any  $n \times n$  matrix  $A$ .
25. For any  $m \times n$  matrix  $A$  it is possible to find an invertible  $m \times m$  matrix  $M$  such that  $MA$  is in row echelon form.
26. The sum of two  $n \times n$  invertible matrices is invertible.
27. If  $A$  and  $B$  are similar matrices, then  $\text{trace}(A) = \text{trace}(B)$ .
28. If  $A$  and  $B$  are similar, then  $\det(A) = \det(B)$ .
29. If two matrices  $A$  and  $B$  have the same characteristic polynomial, then they are similar.
30. If two matrices  $A$  and  $B$  are similar, then they have the same characteristic polynomial.
31. Suppose that  $A \in \mathbb{F}^{n \times m}$ ,  $B \in \mathbb{F}^{m \times p}$ ,  $C \in \mathbb{F}^{p \times r}$ . Then  $(AB)C = A(BC)$ .
32. If  $W_1$  and  $W_2$  are subspaces of  $\mathbb{F}^{n \times 1}$ , then define

$$W_1 \oplus W_2 = \{w_1 + w_2 : w_1 \in W_1, w_2 \in W_2\}.$$

Then  $W_1 \oplus W_2$  is a subspace of  $V$ .

33. Suppose that  $u_1, u_2$  are orthogonal unit vectors in  $\mathbb{R}^{n \times 1}$  and  $c_1, c_2 \in \mathbb{R}$ , then  $\|c_1 u_1 + c_2 u_2\| = \sqrt{c_1^2 + c_2^2}$ .
34. The inverse of a Hermetian matrix is Hermetian.

Answer sheet for true and false.  
Each one is worth 3 points.

10. Circle one: True or False
11. Circle one: True or False
12. Circle one: True or False
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