

There is a total of 100 points. Put your answers on separate sheets of paper. Keep this exam, hand in the answers and all work sheets.

1. (6 points) Let

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -3 & -1 & 2 \\ 1 & 2 & 3 & -1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$$

- (a) What is $A + C$?
- (b) What is $3A$?
- (c) What is AB ?
- (d) What is B^T ?
- (e) What is $\text{col}_3(B)$?
- (f) What is $\text{entry}_{2,3}(B)$?

2. (4 points) (a) What is $I_{4 \times 4}$? (b) What is $Z_{3 \times 5}$?

3. (8 points) Find a matrix R in Echelon form and an invertible matrix M such that $MA = R$ where

$$A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 3 & 6 \\ 1 & 2 \end{bmatrix}.$$

4. (6 points) Consider the following three equations: $x_1 + x_2 + x_3 + x_4 = 1$, $x_2 + x_3 + x_4 = 2$, and $x_4 + x_5 = 3$.

(a) Find matrices A and B such that these three equations are equivalent to the single matrix equation $AX = B$.

(b) Find two solutions of this system such that neither is scalar multiple of the other.

5. (10 points) Find the inverse of the following matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ -1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

6. (4 points) Find a 5×5 elementary matrix E such that for any $5 \times n$ matrix A , EA is the matrix obtained from A by swapping third row and fifth row of A .

7. (10 points) Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix}.$$

Find a 3×3 matrix B which is not the zero matrix Z but such that $AB = Z$.

8. (10 points) Let

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}.$$

Write A^{-1} as the product of elementary matrices.

9. (4 points) Suppose A and B are 5×5 matrices, $\det(A) = 3$ and $\det(B) = 7$.

What is $\det(A^{-1}B^T)$? If it is impossible to be determined by this information, say so.

10. (4 points) Suppose C is a 5×5 matrix with $\det(C) = 3$. What is $\det(2C)$? If it is impossible to be determined by this information, say so.

11. (4 points) Suppose A and B are 5×5 matrices, $\det(A) = 3$ and $\det(B) = -3$. What is $\det(A + B)$? If it is impossible to be determined by this information, say so.

12. (6 points) What is the $\det(A)$ where

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

13. (12 points) Let A be an $m \times n$ matrix, B be an $n \times p$ matrix.

(a) State the definition of $C = AB$.

(b) Let c be a scalar. Prove that $A(cB) = (cA)B$.

14. (12 points) Let A and B be $n \times n$ matrices such that

$$(A + B)^2 = A^2 + 2AB + B^2.$$

Prove that $AB = BA$.

Hint: What is the simplest formula you can get for $(A + B)^2$ without the assumption that $AB = BA$?