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M340 Exam 1
Feb 23, 1990

Put your answers on this exam. Put your work on separate sheets of paper to be handed in with the exam. There are a total of 50 points.

Name_____ Score_____

$$A = \begin{bmatrix} 1 & 1+i & 0 \\ -1 & 2i & 3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1+i \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} i \\ 1 \\ i \end{bmatrix}$$

Compute the following or say that it is undefined.

1. (1 point(s)) $AB=$
2. (1 point(s)) $B + C=$
3. (1 point(s)) $BA=$
4. (1 point(s)) $iA=$
5. (1 point(s)) $A'=$
6. (1 point(s)) $\langle B, C \rangle=$
7. (1 point(s)) $|\langle B, C \rangle|=$
8. (1 point(s)) $\|C\|=$

9. (1 point(s)) If the norm of a vector Q is 6 what is the norm of iQ ?
10. (2 point(s)) If $\langle U, V \rangle = 2 + 3i$ what is $\langle V, 2U \rangle$?
11. (2 point(s)) If B is the inverse of the matrix A , then what is the inverse of the matrix $6A$?
12. (2 point(s)) Give the definition of the inverse of an $n \times n$ matrix A . (Your answer should consist of a grammatically correct sentence.)
13. (2 point(s)) Give an example other than the identity of an orthogonal symmetric 2×2 matrix.
14. (1 point(s)) Give an example of a trivial 3×1 vector.
15. (1 point(s)) Give an example of a nontrivial 2×1 vector.
16. (2 point(s)) For what values of a, b, c does the equation

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} X = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

have

- (a) no solutions?
 - (b) a unique solution?
 - (c) infinitely many solutions?
17. (2 point(s)) What are the slack columns of the matrix A ?

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

18. (4 point(s)) Find the inverse of A or show that the inverse does not exist by exhibiting a nontrivial vector X such that $AX = [0]$. (Remember: Put your work on a separate sheet of paper.)

(a)

$$A = \begin{bmatrix} 0 & -1 & 0 & 2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

(b)

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 3 & 2 \end{bmatrix}$$

19. (2 point(s)) Find an elementary matrix E such that EA is in row echelon form or say that no such E exists.

$$A = \begin{bmatrix} 0 & 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -1 \end{bmatrix}$$

- 20.(1 point)(Circle one: True False.) If U and V are $n \times n$ uppertriangular matrices then so is UV .
- 21.(1 point)(Circle one: True False.) If D is a diagonal $n \times n$ matrix, then so is cD^{n+1} where c is a scalar.
- 22.(1 point)(Circle one: True False.) If A, B, C and D are $n \times n$ matrices, then $(AB)(CD) = A(BC)D$.
- 23.(1 point)(Circle one: True False.) If A and B are $n \times n$ matrices and c is a scalar, then $A(cB) = B(cA)$.
- 24.(1 point)(Circle one: True False.) If A, B , and C are $n \times n$ matrices and $A + C = C + B$, then $A = B$.
- 25.(1 point)(Circle one: True False.) If A is an $n \times n$ invertible matrix, then the matrix equation $AX = B$ has a unique solution.
- 26.(1 point)(Circle one: True False.) If A and B are symmetric $n \times n$ matrices, then AB is a symmetric matrix.
- 27.(1 point)(Circle one: True False.) If A and B are symmetric $n \times n$ matrices, then $A + B$ is a symmetric matrix.
- 28.(1 point)(Circle one: True False.) If A, B , and C are $n \times n$ matrices and $AC = BC$, then either $A = B$ or C is the zero matrix.
- 29.(1 point)(Circle one: True False.) It is possible to find 5×1 vectors U and V such that $\|U\|=2$, $\|V\| = 3$ and $\|U + V\| = 4$.
- 30.(1 point)(Circle one: True False.) It is possible to find 17×1 vectors U and V such that $\|U\|=4$, $\|V\| = 2$ and $\|U + V\| = 7$.
- 31.(1 point)(Circle one: True False.) It is possible to find 3×1 vectors U and V such that $\langle U, V \rangle = 5$ and $\|U\| = \|V\| = 2$.
- 32.(1 point)(Circle one: True False.) For any n there exists an $n \times n$ elementary matrix E such that $EA = A^T$ for any $n \times n$ matrix A .
- 33.(1 point)(Circle one: True False.) For any $m \times n$ matrix A it is possible to find an invertible $m \times m$ matrix M such that MA is in row echelon form.
- 34.(1 point)(Circle one: True False.) The sum of two $n \times n$ invertible matrices is invertible.
35. (6 point(s)) Suppose A is an $n \times n$ matrix such that $\|AX\| = \|X\|$ for all $n \times 1$ vectors X . **On the back of this page** prove that $\langle AX, AY \rangle = \langle X, Y \rangle$ for all $n \times 1$ vectors X and Y .

exam 1 solutions by minimat

```
** Diary file: C:\CLASS\340JAN90\EX1
   opened at Date/Time: 90-02-23 10:38:24**
   (Type  DIARY OFF  to close the diary.)
```

```
#> i=sqrt(-1)
```

```
i =
( 0.000 + 1.000*i)
```

```
#> a=[1 1+i 0; -1 2*i 3]
```

```
a =
( 1.000 + 0.000*i) ( 1.000 + 1.000*i) ( 0.000 + 0.000*i)
( -1.000 + 0.000*i) ( 0.000 + 2.000*i) ( 3.000 + 0.000*i)
```

```
#> b=[0;1+i;1]
```

```
b =
( 0.000 + 0.000*i)
( 1.000 + 1.000*i)
( 1.000 + 0.000*i)
```

```
#> c=[i;1;i]
```

```
c =
( 0.000 + 1.000*i)
( 1.000 + 0.000*i)
( 0.000 + 1.000*i)
```

```
#> a*b
```

```
ans =
```

```

      ( 0.000 + 2.000*i)
      ( 1.000 + 2.000*i)

#> b+c

ans =
      ( 0.000 + 1.000*i)
      ( 2.000 + 1.000*i)
      ( 1.000 + 1.000*i)

#> b*a

[71] : Incompatible sizes for multiplication

#> i*a

ans =
      ( 0.000 + 1.000*i) ( -1.000 + 1.000*i) ( 0.000 + 0.000*i)
      ( 0.000 - 1.000*i) ( -2.000 + 0.000*i) ( 0.000 + 3.000*i)

#> a'

ans =
      ( 1.000 + 0.000*i) ( -1.000 + 0.000*i)
      ( 1.000 - 1.000*i) ( 0.000 - 2.000*i)
      ( 0.000 + 0.000*i) ( 3.000 + 0.000*i)

#> b*c

[71] : Incompatible sizes for multiplication

#> inner(b,c)

ans =
      1.000

```

```
#> norm(c)

ans =
      1.732

#> sqrt(3 )

ans =
      1.732

#> q=6

q =
      6.000

#> norm(i*q)

ans =
      6.000

#> u=2+3*i, v=1

u =
      (  2.000 +  3.000*i)

v =
      1.000

#> inner(u,v)

ans =
      (  2.000 +  3.000*i)

#> inner(v,2*u)
```

```

ans =
      (    4.000 -    6.000*i)

#> a=rand(3,3)

a =
      0.598      0.882      0.295
      0.189      0.486      0.161
      0.476      0.125      0.469

#> b=inv(a)

b =
      3.967     -7.197     -0.027
     -0.225      2.667     -0.774
     -3.966      6.595      2.366

#> inv(6*a)

ans =
      0.661     -1.200     -0.004
     -0.037      0.444     -0.129
     -0.661      1.099      0.394

#> a=[0 -1 0 2; 1 0 0 0 ; 0 0 1 0 ; 0 1 0 -1]

a =
      0.000     -1.000      0.000      2.000
      1.000      0.000      0.000      0.000
      0.000      0.000      1.000      0.000
      0.000      1.000      0.000     -1.000

#> inv(a)

ans =
      0.000      1.000      0.000      0.000
      1.000      0.000      0.000      2.000

```

0.000	0.000	1.000	0.000
1.000	0.000	0.000	1.000

```
#> randsoln(a)
```

```
ans =
```

0.000
0.000
0.000
0.000

```
#> a=[ 1 2 1; 0 1 1; 1 3 2 ]
```

```
a =
```

1.000	2.000	1.000
0.000	1.000	1.000
1.000	3.000	2.000

```
#> inv(a)
```

```
[82] : Vanishing determinant
```

```
#> randsoln(a)
```

```
ans =
```

0.483
-0.483
0.483

```
#> a=[0 1 0 2 0 ; 1 0 0 0 2; 0 0 1 0 -1]
```

```
a =
```

0.000	1.000	0.000	2.000	0.000
1.000	0.000	0.000	0.000	2.000
0.000	0.000	1.000	0.000	-1.000

```
#> [b m]=refm(a)
```

```
b =
```

1.000	0.000	0.000	0.000	2.000
0.000	1.000	0.000	2.000	0.000
0.000	0.000	1.000	0.000	-1.000

```
m =
```

0.000	1.000	0.000
1.000	0.000	0.000
0.000	0.000	1.000

```
#> round(rand(15,1))
```

```
ans =
```

0.000
0.000
0.000
0.000
1.000
0.000
0.000
0.000
0.000
1.000
0.000
0.000
1.000
1.000
0.000
1.000

```
#> quit
```

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