

1. Suppose x, y are two random reals in $[0, 1]$ chosen independently and uniformly. What is the probability that x^2 is less than y ?

Answer: $\frac{2}{3}$

2. Suppose we have two red, two white, and two blue balls in a basket.

Choose 2, if they have the same color you win.

Choose 2 more (without replacing first two), if they have the same color you win.

Finally if the two left in the basket are the same color, you win.

What is the probability that you win?

Answer: $\frac{7}{15}$

3. A box contains 3 cards:

One card has both sides white.

One card has both sides black.

One card has a white side and a black side.

A card is randomly drawn out of the box and placed on a table.

Given that the side facing up is black what is the probability that the other side of the card is black?

Answer: $\frac{2}{3}$

4. Suppose X is a nonnegative random variable and $P(X < 2) \leq \frac{1}{7}$. If $\mu = E(X)$ what is the smallest possible value μ can have?

Answer: $\frac{12}{7}$

5. Let X and Y be random variables. Prove that

$E(XY) = E(X)E(Y)$ if and only if $\text{var}(X + Y) = \text{var}(X) + \text{var}(Y)$.

6. In a long line of children you happen to notice that about 20% of the time a girl is followed by a boy and about 70% of the time a boy is followed by another boy. If you randomly choose a child from the line, what is the probability that its a girl?

Answer: .6

7. Prove $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

8. Suppose a basketball player has a probability of making a 1-point shot of $\frac{1}{2}$ and a 2-point shot of $\frac{1}{3}$. During a game he makes 12 1-point attempts and 27 2-point attempts. Let X be the total number of points he scores. Assuming all shot attempts are pairwise independent Bernoulli trials, find the expected value and variance of X .

Answer: 24, 27