Soyoun Kim

Math 331

2/6/2012

(9) $\lim_{n\to\infty} A_n = B$. $\bigcup_{n=1}^{\infty} \bigcap_{k\ge n} A_k = B = \bigcap_{n=1}^{\infty} \bigcup_{k\ge n} A_k$

Prove that $\lim_{n\to\infty} P(A_n) = P(B)$

9 A

10 A

11 A

a) Let $\bigcap_{k\ge n} A_k = B_n$, then $B_1 \subseteq B_2 \subseteq B_3 \subseteq \cdots \subseteq B_n \subseteq B_{n+1} \subseteq \cdots$

$$P(B) = P\left(\bigcup_{n=1}^{\infty} \bigcap_{k\ge n} A_k\right) = P\left(\bigcup_{n=1}^{\infty} B_n\right) = \lim_{n\to\infty} P(B_n)$$

$$= \lim_{n\to\infty} P\left(\bigcap_{k\ge n} A_k\right) \le \lim_{n\to\infty} P(A_n) \quad \text{becaus } B_n \subseteq A_n$$

b) Let $\bigcup_{k\ge n} A_k = C_n$, $C_n \supseteq C_{n+1}$

$$P(B) = P\left(\bigcap_{n=1}^{\infty} \bigcup_{k\ge n} A_k\right) = P\left(\bigcap_{n=1}^{\infty} C_n\right) = \lim_{n\to\infty} P(C_n)$$

$$= \lim_{n\to\infty} P\left(\bigcup_{k\ge n} A_n\right) \ge \lim_{n\to\infty} P(A_n) \quad \text{becaus } A_n \subseteq C_n$$

From (a) and (b), $\lim_{n\to\infty} P(A_n) \le P(B) \le \lim_{n\to\infty} P(A_n)$

$$\Rightarrow \lim_{n\to\infty} P(A_n) = P(B)$$

(10) Prove that $P(A \triangle B) = P(A) + P(B) - 2P(A\cap B)$

$$P(A \triangle B) = P((A\setminus B) \cup (B \setminus A)) = P(A\setminus B) + P(B\setminus A)$$

($\because$ $(A\setminus B)$ and $(B\setminus A)$ are mutually exclusive)

$$= P(A\setminus(A\cap B)) + P(B\setminus(B\cap A))$$

$$= P(A) - P(A\cap B) + P(B) - P(B\cap A) \quad (\because A \supseteq (A\cap B))$$

$$= P(A) + P(B) - 2P(A\cap B)$$

(11) Let $P(A)$ be the probability that program A is right.

$P(A \cap B) = 0.5$

$P(A^c) = 0.35 = 1 - P(A) \Rightarrow P(A) = 0.65$

$P(B^c) = 0.45 = 1 - P(B) \Rightarrow P(B) = 0.55$

The probability that a randomly chosen student got both programs wrong is $P(A^c \cap B^c)$.

$$P(A^c \cap B^c) = P((A \cup B)^c) = 1 - P(A \cup B)$$
$$= 1 - \{P(A) + P(B) - P(A \cap B)\}$$
$$= 1 - \{0.65 + 0.55 - 0.5\}$$
$$= 0.3$$