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Math 331
HW 79
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(79) Calculate the Lebesgue measure of the set

$$A = \bigcup_{n=1}^{\infty} \left(n! + \frac{1}{3^n}, n! + \frac{1}{2^n} \right]$$

* a disjoint union of sets

$$B = \left(n! + \frac{1}{3^n}, n! + \frac{1}{2^n} \right]$$

A

$$\lambda(B) = n! + \frac{1}{2^n} - n! - \frac{1}{3^n} = \frac{1}{2^n} - \frac{1}{3^n} = \left(\frac{1}{2}\right)^n - \left(\frac{1}{3}\right)^n$$

$$\lambda(A) = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n - \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n = \frac{1/2}{1 - 1/2} - \frac{1/3}{1 - 1/3} = 1 - \frac{1}{2}$$

$$= \frac{1}{2}$$