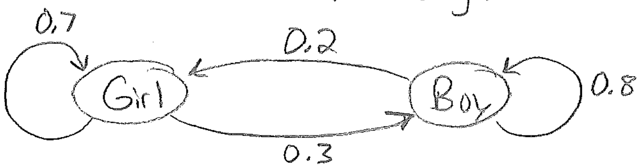
Math 331 | 5/2/12 | Jake Darlington

5-2-76 In a long line of children you notice that about 30% of the time a girl is followed by a boy and about 80% of the time a boy is followed by another boy. If you randomly choose a child from the line, what is the probability that its a girl?



$$\Rightarrow P = \begin{array}{c c} & \begin{array}{cc} G & B \end{array} \\ \begin{array}{c} G \\ B \end{array} & \begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix} \end{array}$$

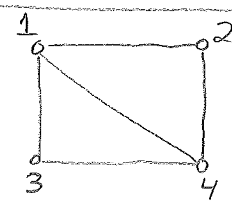
$$[x \quad y]\begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix} = [x \quad y] \Rightarrow \begin{cases} 7x + 2y = 10x \\ 3x + 8y = 10y \end{cases} \qquad x + y = 1$$

76 A
77 A
78 A

$$\Rightarrow \begin{cases} -3x + 2y = 0 \\ 3x - 2y = 0 \end{cases} \Rightarrow x = \tfrac{2}{3}y \qquad \begin{array}{l} x + y = 1 \\ \tfrac{2}{3}y + y = 1 \Rightarrow y = \tfrac{3}{5} \Rightarrow x = \tfrac{2}{5} \end{array}$$

$$\vec{S} = [0.4 \quad 0.6] \Rightarrow \boxed{\text{Probability that its a girl} = \underline{\underline{0.4}}}$$

5-2-77 For random walk on the graph:



Find steady state prob. vector.

$$P = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1/3 & 1/3 & 1/3 \\ 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \\ 1/3 & 1/3 & 1/3 & 0 \end{bmatrix} \end{array} \qquad \vec{S} = [x \quad y \quad z \quad w] \qquad x + y + z + w = 1$$

$$\vec{S}P = \vec{S} \Rightarrow [x\ y\ z\ w]\begin{bmatrix} 0 & 1/3 & 1/3 & 1/3 \\ 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \\ 1/3 & 1/3 & 1/3 & 0 \end{bmatrix} = [x\ y\ z\ w] \Rightarrow \begin{cases} \frac{1}{2}(y+z) + \frac{1}{3}w = x \\ \frac{1}{3}(x+w) = y \\ \frac{1}{3}(x+w) = z \\ \frac{1}{3}x + \frac{1}{2}(y+z) = w \end{cases} \Rightarrow \begin{cases} 3y + 3z + 2w = 6x \\ x + w = 3y \\ x + w = 3z \\ 2x + 3y + 3z = 6w \end{cases}$$

$$\Rightarrow x + w = 3y = 3z \Rightarrow y = z$$

$$\Rightarrow \begin{cases} 3y + 3z + 2w - 6x = 0 \\ -(3y + 3z - 6w + 2x = 0) \end{cases}$$
$$8w - 8x = 0$$
$$\Rightarrow w = x$$

$$x + w = 3y$$
$$2x = 3y$$
$$\Rightarrow x = \tfrac{3}{2}y$$

$$\Rightarrow x + y + z + w = 1$$
$$\tfrac{3}{2}y + y + y + \tfrac{3}{2}y = 1$$
$$\Rightarrow y = 1/5 \Rightarrow z = 1/5$$
$$x = 3/10 \Rightarrow w = 3/10$$

$$\Rightarrow \boxed{\vec{S} = [0.3 \quad 0.2 \quad 0.2 \quad 0.3]}$$

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5-2-78 Suppose P is the transition matrix for an ergodic Markov chain. Let $Q = \frac{1}{2}I + \frac{1}{2}P$

a) Prove Q is regular

b) Prove that if $\vec{s}Q = \vec{s}$, then $\vec{s}P = \vec{s}$

a) Prove that Q is regular

P is transition matrix for ergodic Markov chain $\Rightarrow$ ergodic means there is a positive prob. of going from one state to another in some number of steps

$\Rightarrow$ hence P has no absorbing states and has positive probabalities of moving from one state to another in some # of steps

$\Rightarrow Q = \frac{1}{2}(I+P)$ definitely has positive entries for q_{ii} (on the main diagonal) because of the identity matrix. As P is ergodic, entries p_{ij} for some number of steps $\Rightarrow$ q_{ij} are positive for some # of steps

$\Rightarrow$ therefore q_{ii} and q_{ij} will be positive for some number of steps $\Rightarrow$ hence Q is regular

b) Prove that if $\vec{s}Q = \vec{s}$, then $\vec{s}P = \vec{s}$

As $Q = \frac{1}{2}I + \frac{1}{2}P$ $\Rightarrow$ $\vec{s}Q = \vec{s}$ $\Rightarrow$ $\vec{s}\left(\frac{1}{2}I + \frac{1}{2}P\right) = \vec{s}$

$\Rightarrow \frac{1}{2}\vec{s}I + \frac{1}{2}\vec{s}P = \vec{s}$

I is identity, so $\vec{s}I = \vec{s}$

$\Rightarrow \frac{1}{2}\vec{s} + \frac{1}{2}\vec{s}P = \vec{s}$

$\Rightarrow \cancel{\tfrac{1}{2}}\vec{s}P = \cancel{\tfrac{1}{2}}\vec{s}$

$\Rightarrow \vec{s}P = \vec{s}$

$\Rightarrow$ $\boxed{\text{Hence if } \vec{s}Q = \vec{s}, \text{ then } \vec{s}P = \vec{s}}$

Brian Murray

78 a Since P is ergotic, it is possible to go from any state (i) to any state (j) in some number of steps. By adding the identity matrix, it becomes possible to go from any state to itself in one step. Thus, if you find the longest path in P, You will be able to reach all other states in that number of steps by continually looping in one state until leaving will reach the end state in that number of steps.

OK