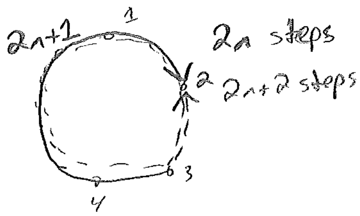
Math 331	4/30/12	Jake Darlington

4-30-75 Prove that a random walk on an n-cycle is regular iff n is odd.

A

1) If n is odd $\Rightarrow$ regular

Take random walk on n-cycle where n is odd. A markov chain is regular if its possible with positive probability to go from any state (i) to another state (j) in K-steps.

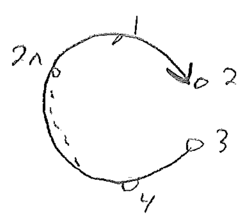


$\Rightarrow$ There exists a K such that K is $2n + 2n+2$ steps in moving from any state (i) to another state (j) $\Rightarrow$ hence if 'n = odd' $\Rightarrow$ regular

Alternatively, state (i) can be either an even or odd-numbered state, and then it is possible to go to an even or odd state (j) in K steps. Hence it is possible with positive probability to go from state (i) to state (j) in K steps

$\Rightarrow$ hence if n-cycle is odd $\Rightarrow$ it is regular

2) If n is even $\Rightarrow$ not regular



If n is even and state (i) is an even numbered state, then it is impossible to go to an odd-numbered state (j) if K is an even number of steps.

Likewise, if state (i) is odd, then it is impossible to go to an ~~even~~ odd numbered state (j) if K is an odd number of steps.

Hence there does not exist a K where its possible to go from state (i) to (j) with positive probability in K steps.

$\Rightarrow$ If n-cycle is even $\Rightarrow$ it is not regular

$\Rightarrow$ Hence a random walk on an n-cycle is regular iff n is odd.