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Hw 63-64
Math 381

(63) Suppose you have 6 RED and 6 BLACK balls in a basket. Consider the following algorithm.

(1) choose a random ball
(2) if RED place it back into the basket — 63 A
(3) if BLACK remove it from the basket — 64 A
(4) if there are more BLACK balls GOTO (1)
(5) STOP

Find $E(X)$ where $X$ = # times step 1 is executed.

Let $X = Y_1 + Y_2 + Y_3 + Y_4 + Y_5 + Y_6$, where $Y_i$ = # of times line 1 is executed until the i-th Black ball is chosen

Geometric Expectation Formula:

$$E(H) = \frac{1}{p}$$

$$E(X) = E(Y_1) + E(Y_2) + E(Y_3) + E(Y_4) + E(Y_5) + E(Y_6)$$

$$= \frac{1}{\frac{6}{12}} + \frac{1}{\frac{5}{11}} + \frac{1}{\frac{4}{10}} + \frac{1}{\frac{3}{9}} + \frac{1}{\frac{2}{8}} + \frac{1}{\frac{1}{7}}$$

$$= \frac{207}{10} \approx 20.7$$

(64) Tom, Dick, and Harry flip a silver dollar repeatedly until one of them gets a head and wins the dollar.

Order of tosses is T, D, H, T, D, H, T...

Find the expected value of the amount each win

$p(\text{HEADS}) = p$, $0 < p < 1$

$p(\text{TAILS}) = q$, $p + q = 1$

What are these three values when $p = \frac{1}{2}$? what happens as $p \to 0$?

$$P(\text{Tom Wins}) = P(\text{HEADS}) + P(\text{TAILS}^3\,\text{HEADS}) + P(\text{TAILS}^6\,\text{HEADS}) + \cdots$$
$$= p + q^3 p + q^6 p + \cdots$$
$$= \sum_{n=0}^{\infty} p \cdot q^{3n} = p \cdot \sum_{n=0}^{\infty} (q^3)^n = p \cdot \frac{1}{1-q^3} = \frac{p}{1-q^3}$$

$$P(\text{Dick Wins}) = P(\text{TAILS HEADS}) + P(\text{TAILS}^4\,\text{HEADS}) + P(\text{TAILS}^7\,\text{HEADS}) + \cdots$$
$$= \sum_{n=0}^{\infty} p \cdot q^{3n+1} = pq \cdot \sum_{n=0}^{\infty} (q^3)^n = \frac{pq}{1-q^3}$$

$$P(\text{Harry Wins}) = P(\text{TAILS}^2\,\text{HEADS}) + P(\text{TAILS}^5\,\text{HEADS}) + P(\text{TAILS}^8\,\text{HEADS}) + \cdots$$
$$= \frac{pq^2}{1-q^3}$$

$$E(T) = 0 + 1 \cdot \frac{p}{1-q^3} = \boxed{\frac{p}{1-q^3}}, \quad p = \tfrac{1}{2} \Rightarrow q = \tfrac{1}{2} \quad \frac{\frac{1}{2}}{1-\left(\frac{1}{2}\right)^3} = \boxed{\frac{4}{7}}$$

$$E(D) = 0 + 1 \cdot \frac{pq}{1-q^3} = \boxed{\frac{pq}{1-q^3}}, \quad p = \tfrac{1}{2} \Rightarrow q = \tfrac{1}{2} \quad \frac{\left(\frac{1}{2}\right)^2}{1-\left(\frac{1}{2}\right)^3} = \boxed{\frac{2}{7}}$$

$$E(H) = \frac{pq^2}{1-q^3} = \boxed{\frac{1}{7}}$$