zach knoeck

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad \sum_{n=1}^{\infty} \frac{q^{n-1}}{n!} = \sum_{n=0}^{\infty} \frac{q^{n-1}}{n!} - \frac{1}{q}$$

HW For Friday

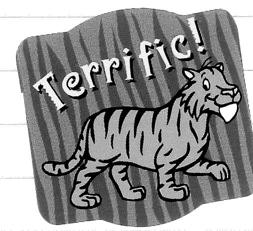
60A
61A

3-30-60 Given $0<p<1$ and $p+q=1$, Suppose that for every $n=1,2,\ldots$ $P(X=n)=q^{n-1}p$ find $E\left(\frac{1}{X!}\right)$

$$E\left(\frac{1}{X!}\right) = \sum_{n=1}^{\infty} \frac{1}{n!} P(X=n) = \sum_{n=1}^{\infty} \frac{1}{n!} q^{n-1} p = p \sum_{n=1}^{\infty} \frac{q^{n-1}}{n!} =$$

$$E = p\left(\sum_{n=0}^{\infty} \frac{q^{n-1}}{n!} - \frac{1}{q}\right) = p\left(\sum_{n=0}^{\infty} \frac{q\,q^{n-1}}{q\,n!} - \frac{1}{q}\right) = p\left(\sum_{n=0}^{\infty} \frac{q^n}{q\,n!} - \frac{1}{q}\right)$$

$$= \frac{p}{q}\left(\sum_{n=0}^{\infty} \frac{q^n}{n!} - 1\right) = \boxed{\frac{p}{q}\left(e^q - 1\right)}$$



3-30-61 Give an example of a discrete random variable $X \geq 0$ such that $E(X)$ is finite but $E(X^2)=\infty$

Hint: $\underbrace{\sum_{n=1}^{\infty} \frac{1}{n} = \infty}_{E(X^2)} \qquad \underbrace{\sum_{n=1}^{\infty} \frac{1}{n^2} < \infty}_{E(X)}$

$\sum X_n P(X=n) < \infty$ $\quad \frac{1}{n^2}$

$\sum (X_n)^2 P(X=n) < \infty$ $\quad \frac{1}{n}$

$X_n = n \quad P = \frac{1}{n^3\ldots}$

$\frac{n}{n^3} \; \frac{1}{n^2} \checkmark < \infty$

$\frac{n^2}{n^3} = \frac{1}{n} \checkmark = \infty$

$$\sum_n P(X=n) = 1 \qquad \sum_{n=1}^{\infty} \frac{1}{n^3\ldots} = 1$$

riemans zeta function

$$\sum_{n=1}^{\infty} \frac{1}{n^3} = \mathrm{zeta}(3) \approx 1.202\ldots \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^3\,\mathrm{zeta}[3]} = 1$$

$$X_n = n \qquad P = \frac{1}{n^3\,\mathrm{zeta}[3]}$$

$Z[3] = \mathrm{zeta}[3]$

$$E(X) = \sum_{n=1}^{\infty} (n)\left(\frac{1}{n^3\,\mathrm{zeta}[3]}\right) = \sum_{n=1}^{\infty} \frac{1}{n^2} \cdot \frac{1}{\mathrm{zeta}[3]} = \frac{1}{Z[3]} \sum_{n=1}^{\infty} \frac{1}{n^2} < \infty \checkmark$$

constant pullout

does not converge

$$E(X^2) = \sum_{n=1}^{\infty} n^2 \frac{1}{n^3 Z[3]} = \frac{1}{Z[3]} \sum \frac{1}{n} = \frac{1}{Z[3]} \infty = \infty \checkmark$$

for $n=1,2,\ldots$ $\quad X_n = n \quad P(X=n) = \frac{1}{n^3\,\mathrm{zeta}[3]}$