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Math 331

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58 A
59 A

$P(X=0) = P(X=1) = 0$.

58. $P(X=k) = b^{k-1}g + g^{k-1}b \qquad k = 2, 3, \cdots, n, n+1, \cdots$

$$E(X) = \sum_{n=2}^{\infty} n \cdot \left(b^{n-1}g + g^{n-1}b\right) \qquad \sum_{n=1}^{\infty} n x^{n-1} = \frac{1}{(1-x)^2}$$

$$= g\sum_{n=1}^{\infty} n b^{n-1} - g + b\sum_{n=1}^{\infty} n g^{n-1} - b$$

$$= \frac{g}{(1-b)^2} + \frac{b}{(1-g)^2} - (g+b)$$

$$= \frac{1}{g} + \frac{1}{b} - 1$$

$$Var(X) = E(X^2) - E^2(X)$$

$$E(X^2) = \sum_{n=2}^{\infty} n^2\left(b^{n-1}g + g^{n-1}b\right) \qquad \sum_{n=1}^{\infty} n^2 x^{n-1} = \frac{1+x}{(1-x)^3}$$

$$= g\frac{1+b}{(1-b)^3} + b\frac{1+g}{(1-g)^3} - 1$$

$$= \frac{1+b}{g^2} + \frac{1+g}{b^2} - 1 \qquad \frac{2}{g} + \frac{2}{b}$$

$$Var(X) = \frac{1+b}{g^2} + \frac{1+g}{b^2} - 1 - \frac{1}{g^2} - \frac{1}{b^2} - 1 + \frac{2}{g} + \frac{2}{b} - \frac{2}{gb}$$

$$= \frac{b}{g^2} + \frac{g}{b^2} - 2$$

$$\sigma(X) = \sqrt{\frac{b}{g^2} + \frac{g}{b^2} - 2}$$

59. $b=\frac{1}{2}$ $g=\frac{1}{2}$

$$E(X) = \frac{1}{g} + \frac{1}{b} - 1 = 2+2-1=3$$

$$Var(X) = \frac{b}{g^2} + \frac{g}{b^2} - 2 = 2+2-2=2$$

$$\sigma(X) = \sqrt{2}$$

$b=\frac{1}{3}$ $g=\frac{2}{3}$

$$E(X) = \frac{1}{g} + \frac{1}{b} - 1 = 3 + \frac{3}{2} - 1 = \frac{7}{2}$$

$$Var(X) = \frac{b}{g^2} + \frac{g}{b^2} - 2$$

$$= \frac{\frac{1}{3}}{\left(\frac{2}{3}\right)^2} + \frac{\frac{2}{3}}{\left(\frac{1}{3}\right)^2} - 2$$

$$= \frac{3}{4} + 6 - 2$$

$$= \frac{19}{4}$$

$$\sigma(X) = \sqrt{\frac{19}{4}} = \frac{\sqrt{19}}{2}$$