Math 331 | 3/26/12 | Jake Darlington

$\boxed{3\text{-}26\text{-}56}$

Casino randomly chooses 20 out of 80

Mark 5

Catch	Bally's \$10 bet pays out	Barbary \$1 bet pays out
3	10	1
4	200	25
5	5500	600

Find $E(X)$, $E(Y)$

X = amount Bally will pay out on a \$10 bet where player marked 5 spots
Y = amount Barbary will pay out on a \$1 bet where player marked 5 spots

$C=20$ $N=80$

marked = 20
not-marked = 80 − 20 = 60

Catch	X (payout)	P(X=x) / P(Y=y)	Y	XP(X=x)	YP(Y=y)
0	0	$\frac{\binom{20}{0}\binom{60}{5}}{\binom{80}{5}} = 0.22718$	0	0	0
1	0	$\frac{\binom{20}{1}\binom{60}{4}}{\binom{80}{5}} = 0.40569$	0	0	0
2	0	$\frac{\binom{20}{2}\binom{60}{3}}{\binom{80}{5}} = 0.27046$	0	0	0
3	10	$\frac{\binom{20}{3}\binom{60}{2}}{\binom{80}{5}} = 0.08394$	1	0.8394	0.08394
4	200	$\frac{\binom{20}{4}\binom{60}{1}}{\binom{80}{5}} = 0.01209$	25	2.418	0.30225
5	5500	$\frac{\binom{20}{5}\binom{60}{0}}{\binom{80}{5}} = 0.00064$	600	3.52	0.384
Sum		1		6.78	0.77

56 A
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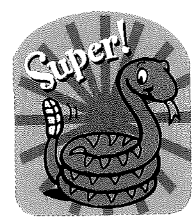
$E(X) = \sum_1^n xP(X=x) \Rightarrow \boxed{E(X) = \$6.78}$

$E(Y) = \sum_1^n yP(Y=y) \Rightarrow \boxed{E(Y) = \$0.77}$



Let Z_K be the number of catches for a card with K spots marked out of a possible 80. Find $E(Z_K)$ and $Var(Z_K)$ for K = 5, 6, 7.

$\Rightarrow$ n=80, casino chooses 20 : $p = \frac{20}{80} = \frac{1}{4}$

$E(Z_K) = Kp = \frac{K}{4}$ $\qquad Var(Z_K) = Kp(1-p)\left[1 - \frac{K-1}{N-1}\right] = K\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\left[1 - \frac{K-1}{79}\right]$

$E(Z_5) = \frac{5}{4} = 1.25$ $\qquad Var(Z_5) = 5\left(\frac{3}{16}\right)\left[1 - \frac{4}{79}\right] = 0.890$

$E(Z_6) = \frac{6}{4} = 1.5$ $\qquad Var(Z_6) = 6\left(\frac{3}{16}\right)\left[1 - \frac{5}{79}\right] = 1.054$

$E(Z_7) = \frac{7}{4} = 1.75$ $\qquad Var(Z_7) = 7\left(\frac{3}{16}\right)\left[1 - \frac{6}{79}\right] = 1.213$

$\Rightarrow$

K	$E(Z_K)$	$Var(Z_K)$
5	1.25	0.890
6	1.5	1.054
7	1.75	1.213

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Beeno → like Keno, but uses binomial distribution.
Casino chooses each number independently with prob. 1/4.
Repeat problem 56 for analagous variables X', Y', Z_K'.

$n = 80,\ p = 1/4 \rightarrow BIN(80, 1/4)$ BIN: $P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$

Catch	X'	Y'	$P(X'=x)$ / $P(Y'=y)$	$X'P(X'=x)$	$Y'P(Y'=y)$
0	0	0	$\binom{5}{0}\left(\frac{1}{5}\right)^0\left(\frac{3}{4}\right)^5 = 0.23730$	0	0
1	0	0	$\binom{5}{1}\left(\frac{1}{5}\right)^1\left(\frac{3}{4}\right)^4 = 0.39551$	0	0
2	0	0	$\binom{5}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3 = 0.26367$	0	0
3	10	1	$\binom{5}{3}\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2 = 0.08789$	0.8789	0.08789
4	200	25	$\binom{5}{4}\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^1 = 0.01465$	2.9297	0.3662
5	5500	600	$\binom{5}{5}\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^0 = 0.00098$	5.3711	0.5859
Sum			1	9.18	1.04

$E(X') = \sum_{i}^{n} X'P(X'=x) \Rightarrow \boxed{E(X') = \$9.18}$

$E(Y') = \sum_{i}^{n} Y'P(Y'=y) \Rightarrow \boxed{E(Y') = \$1.04}$

Let Z_K' be the number of catches for a card with K spots marked out of a possible 80.
Find $E(Z_K')$, $Var(Z_K')$

$\Rightarrow\ p = \frac{1}{4}$

$E(Z_K') = Kp$ $\qquad$ $Var(Z_K') = Kp(1-p) = K\left(\frac{1}{4}\right)\left(\frac{3}{4}\right) = \frac{3}{16}K$

$E(Z_5') = 5/4 = 1.25$ $\qquad$ $Var(Z_5') = \frac{3}{16}(5) = \frac{15}{16} = 0.9375$

$E(Z_6') = 6/4 = 1.5$ $\qquad$ $Var(Z_6') = \frac{3}{16}(6) = \frac{18}{16} = 1.125$

$E(Z_7') = 7/4 = 1.75$ $\qquad$ $Var(Z_7') = \frac{3}{16}(7) = \frac{21}{16} = 1.3125$

$\Rightarrow$

K	$E(Z_K')$	$Var(Z_K')$
5	1.25	0.9375
6	1.5	1.125
7	1.75	1.3125