

Math 331

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#44

$$(a) E(X) = a \cdot \varepsilon + \varepsilon \cdot 1 = (a+1)\varepsilon$$

$$E(Y) = \varepsilon \cdot 1 + b\varepsilon = \varepsilon(b+1)$$

$$E(X)E(Y) = \varepsilon^2(a+1)(b+1)$$

~~$E(XY) = \varepsilon^2(a+b)$~~

$$E(XY) = a \cdot \varepsilon + b \cdot \varepsilon = \varepsilon(a+b)$$

44 A

45 A

46 A

b) Let $\varepsilon = \frac{1}{4}$, $a = 0$, $b = \frac{1}{3}$.

$$E(X)E(Y) = \frac{1}{16} (0+1) (1+\frac{1}{3}) = \frac{1}{12} = \frac{1}{4} (0+\frac{1}{3}) = E(XY)$$

#45

a) $\text{Var}(X) = E(X^2) - E(X)^2 = 3 - 4 = -1 < 0$

\Rightarrow impossible

b) Let $P(X=-1) = \frac{1}{3}$, $P(X=2) = \frac{2}{3}$.

$$E(X) = -1 \cdot \frac{1}{3} + 2 \cdot \frac{2}{3} = 1$$

$$E(X^2) = (-1)^2 \cdot \frac{1}{3} + 2^2 \cdot \frac{2}{3} = 3$$

#46

$$0 = \text{Var}(X+Y) - \text{Var}(X) - \text{Var}(Y)$$

$$= E((X+Y)^2) - E(X+Y)^2 - E(X^2) + E(X)^2 - E(Y^2) + E(Y)^2$$

$$= E(X^2 + 2XY + Y^2) - (E(X) + E(Y))^2 - E(X^2) + E(X)^2 - E(Y^2) + E(Y)^2$$

$$= E(X^2) + 2 \cdot E(XY) + E(Y^2) - E(X)^2 - 2E(X)E(Y) - E(Y)^2 - E(X^2) + E(X)^2 - E(Y^2) + E(Y)^2$$

$$= 2E(XY) - 2E(X)E(Y)$$

$$\Rightarrow E(XY) = E(X)E(Y)$$

