Math 331 | 3/12/12 | Jake Darlington

3-12-41 Russian Roulette: The Master of Ceremonies puts one bullet into a six shooter and randomly rotates the chamber. There are six players who one-by-one take the pistol to their head and squeeze trigger. X is the player that shoots himself. Find $E(X)$, $Var(X)$, $\sigma(X)$.

41 A
42 A
43 A

e.g throw dice once, X is number showing $\longrightarrow$ equal probability $X=1, X=2, \ldots X=6$.

X	$P(X=x)$	$xP(X=x)$	X^2	$X^2P(X=x)$
1	1/6	1/6	1	1/6
2	1/6	2/6	4	4/6
3	1/6	3/6	9	9/6
4	1/6	4/6	16	16/6
5	1/6	5/6	25	25/6
6	1/6	6/6	36	36/6
sum	1	21/6		91/6

$$E(X) = \sum_x xP(X=x) = \frac{21}{6} \longrightarrow \boxed{E(X) = 3.5}$$

$$Var(X) = E(X^2) - (E(X))^2 = \frac{91}{6} - \left(\frac{21}{6}\right)^2 = 15.167 - 12.25 = 2.917$$

$$\Rightarrow \boxed{Var(X) = 2.917}$$

$$\sigma = \sqrt{v} = \sqrt{2.917} \Rightarrow \boxed{\sigma = 1.708}$$

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3-12-42 Russian Roulette Variation

MC puts one bullet into sixshooter and randomly rotates chamber. He gives it to player 1, who pulls trigger. If shot, game is over. Otherwise rotate chamber, hand to player 2.
Y = player that is shot. $Y=0 \rightarrow$ all players survive. Find $E(Y)$, $Var(Y)$, $\sigma(Y)$

each turn $\rightarrow p = \frac{1}{6}$ player is shot

$(1-p) \rightarrow$ probability previous player survives

Y	$P(Y=y)$
1	$\frac{1}{6}$
2	$\frac{5}{6}\left(\frac{1}{6}\right) = \frac{5}{36}$ ← 1st player survives, 2nd player dies
3	$\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right) = \frac{25}{216}$
4	$\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) = \frac{125}{1296}$
5	$\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right) = \frac{625}{7776}$
6	$\left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right) = \frac{3125}{46656}$
0	$\left(\frac{5}{6}\right)^6 = \frac{15625}{46656}$

Y	$P(Y=y)$	$YP(Y=y)$	Y^2	$Y^2P(Y=y)$
1	$1/6$	$1/6$	1	$1/6$
2	$5/36$	$5/18$	4	$5/9$
3	$25/216$	$25/72$	9	$25/24$
4	$125/1296$	$125/324$	16	$125/81$
5	$\frac{625}{7776}$	$\frac{3125}{7776}$	25	$\frac{15625}{7776}$
6	$\frac{3125}{46656}$	$\frac{3125}{7776}$	36	$\frac{3125}{1296}$
0	$\frac{15625}{46656}$	0	0	0
Sum	1	1.9812		7.7278

$E(Y) = \sum_y yP(Y=y) \Rightarrow \boxed{E(Y) = 1.981}$

$Var(Y) = E(Y^2) - (E(Y))^2 = 7.7278 - (1.9812)^2 = 3.8026 \Rightarrow \boxed{Var(Y) = 3.803}$

$\sigma = \sqrt{V} = \sqrt{3.803} \Rightarrow \boxed{\sigma = 1.95}$

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3-12-43 | X, Y in same probability space.

X	$P(X=x)$	$xP(X=x)$
2	$1/3$	$2/3$
3	$1/3$	1
6	$1/3$	2

Y	$P(Y=y)$	$YP(Y=y)$
1	$1/4$	$1/4$
2	$1/4$	$1/2$
3	$1/2$	$3/2$

a) $E(X) = \sum_x xP(X=x) = \frac{2}{3} + 1 + 2 = \frac{11}{3} \Rightarrow \boxed{E(X) = 3.67}$

$E(Y) = \sum_y yP(Y=y) = \frac{1}{4} + \frac{1}{2} + \frac{3}{2} = \frac{9}{4} \Rightarrow \boxed{E(Y) = 2.25}$

$E(X)E(Y) = \left(\frac{11}{3}\right)\left(\frac{9}{4}\right) = \frac{33}{4} \Rightarrow \boxed{E(X)E(Y) = 8.25}$

b) Suppose X and Y are independent $\rightarrow$ compute $E(XY)$

independent $\rightarrow$ $E(XY) = E(X)E(Y) = \left(\frac{11}{3}\right)\left(\frac{9}{4}\right) = \frac{33}{4} \Rightarrow$ $\boxed{E(XY) = 8.25}$ if X, Y independent

c) Suppose $P(X=a \text{ and } Y=b) = \frac{1}{9}$ for all a, b except: $P(X=6 \text{ and } Y=1) = \frac{1}{4} - \frac{2}{9}$

$P(X=6 \text{ and } Y=2) = 1/4 - 2/9$

$P(X=6 \text{ and } Y=3) = 1/2 - \frac{2}{9}$

Compute $E(XY)$

$$E(XY) = \sum_c cP(XY=c) = \sum_{a,b} ab\,P(X=a \text{ and } Y=b)$$

$$= (2)(1)\left(\tfrac{1}{9}\right) + (2)(2)\left(\tfrac{1}{9}\right) + (2)(3)\left(\tfrac{1}{9}\right) + (3)(1)\left(\tfrac{1}{9}\right) + (3)(2)\left(\tfrac{1}{9}\right) + (3)(3)\left(\tfrac{1}{9}\right)$$

$$+ (6)(1)\left(\tfrac{1}{4} - \tfrac{2}{9}\right) + (6)(2)\left(\tfrac{1}{4} - \tfrac{2}{9}\right) + (6)(3)\left(\tfrac{1}{2} - \tfrac{2}{9}\right)$$

$$= \frac{1}{9}\left[2 + 4 + 6 + 3 + 6 + 9\right] + 6\left[\left(\frac{1}{36}\right) + 2\left(\frac{1}{36}\right) + 3\left(\frac{5}{18}\right)\right]$$

$$= \frac{1}{9}(30) + 6\left(\frac{11}{12}\right) = \frac{53}{6}$$

$\Rightarrow \boxed{E(XY) = 8.83}$ if X, Y not independent, with probabilities given above