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M331
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Homework 2012-03-05

M Mar 5 **(36)** Three distinguishable dice are thrown. The random variable X is the number of different numbers that appear, e.g., $X(\langle 5,3,5\rangle) = 2$. Find $E(X)$.

The random variable X can take on the values $1, 2, 3$. Given that $E(X) = \sum_{x \in X} xP(X = x)$, solve for the probabilities of each $x \in 1, 2, 3$.

$X = 1$:
This means all of the numbers are the same between the three dice. The number of these are:
$6 \times 1 \times 1$,
as there are six possibilities for the first die, and only one for the second and third.

$X = 2$:
This means two of the numbers are the same, the other two are different. The number of these are:
$\binom{3}{2} \times 6 \times 5$,
as there are $\binom{3}{2}$ ways to select two of the three dice to be identical, six possible values of these, and five of the remaining die.

$X = 3$:
This means all of the three dice take on different values. This is equivalent to:
$6 \times 5 \times 4$.

The total possible sample space size is: $6 \times 6 \times 6$. To confirm that the counts add up to the total sample space size, let:

$$
\begin{aligned}
|S| &= 6 \times 6 \times 6 \\
&= |\{\omega : X(\omega) = 1\}| + |\{\omega : X(\omega) = 2\}| + |\{\omega : X(\omega) = 3\}| \\
&= 6 + \binom{3}{2} \times 6 \times 5 + 6 \times 5 \times 4 \\
&= 6 \times (1 + \binom{3}{2} \times 5 + 5 \times 4) \\
&= 6 \times 36 = 6 \times 6 \times 6
\end{aligned}
$$

Hence:

$$
\begin{aligned}
E(X) &= (1) \times \frac{6}{6^3} + (2) \times \frac{\binom{3}{2} \times 6 \times 5}{6^3} + (3) \times \frac{6 \times 5 \times 4}{6^3} \\
&= (1) \times \frac{1}{36} + (2) \times \frac{15}{36} + (3) \times \frac{20}{36} \\
&= \frac{1 + 30 + 60}{36} = \frac{91}{36} \\
&= 2.528
\end{aligned}
$$