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math 331
3/2/12

34) Assume A, B indep and A, C indep. Prove that
A, $B \cup C$ are indep iff A, $B \cap C$ are indep.

Pf: 34 A
35

($\Rightarrow$): A, $B \cup C$ indep implies A, $B \cap C$ indep

$P(A \cap (B \cup C)) = P(A) P(B \cup C)$ ... (indep)

and $P(A \cap (B \cup C)) = P((A \cap B) \cup (A \cap C))$ (distr.)

$P(A) P(B \cup C) = P((A \cap B) \cup (A \cap C))$

$P(A)[P(B) + P(C) - P(B \cap C)] = P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$

$P(A)P(B) + P(A)P(C) - P(A)P(B \cap C) = P(A)P(B) + P(A)P(C) - P(A \cap B \cap C)$

$P(A) P(B \cap C) = P(A \cap B \cap C)$

This is the def. of independence,
so A, $B \cap C$ are indep.

($\Leftarrow$): A, $B \cap C$ are indep implies A, $B \cup C$ are indep.

$P(A \cap (B \cup C)) = P((A \cap B) \cup (A \cap C))$
$= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$
$= P(A)P(B) + P(A)P(C) - P(A)P(B \cap C)$ indep. assump's
$= P(A)[P(B) + P(C) - P(B \cap C)]$
$= P(A) P(B \cup C)$ (from $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$)

so A, $B \cup C$ are indep.

35) If you randomly choose one of the following answers, what is the probability you've chosen the right answer?

a) 25%
b) 50%
c) 50%
d) 100%

If we only care about the letter chosen rather than the value (e.g., if answering b is right but answering c is considered wrong), then there is a 1/4 chance of picking the right answer. (Ans=a)

If we care only about the value and not the letter (ie, both b and c are accepted) then there are 2 correct ways to answer, so $2/4 = 1/2 = 50\%$. (Ans=b or c)

If we consider the union of these events, then a is correct, b and c are correct, so d is also correct (ie, no way to pick a wrong answer). This is the situation observed in some humanities classes, where any answer is right if supported by the proper argument. (Ans=d; or a, b, c w/ correct arg)