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M 331
Homework

2-17-20 Suppose an urn contains 6 red, 6 blue, and 6 white balls and we simultaneously randomly choose 3 balls from the urn.

A

Treat the balls as all distinct, numbered from 1 to 18. There are $\binom{18}{3}$ ways of choosing 3 balls from 18 distinct ones.

$|U| = \binom{18}{3} = 816$

a) What is the probability ~~that~~ none of the three are red?

How many cases are there where there no red balls? Call this set A.

1. All blue : $\binom{6}{3}$ ways of choosing 3 distinct blue balls
2. All white : $\binom{6}{3}$ " " " " " white "
3. Two blue and one white : $\binom{6}{2} \cdot 6$
4. Two white and one blue : $\binom{6}{2} \cdot 6$

$$|A| = \binom{6}{3} + \binom{6}{3} + \binom{6}{2} \cdot 6 + \binom{6}{2} \cdot 6 = 220$$

$$Pr(\text{All three red}) = \frac{220}{816} \cong 0.2696$$ OK

b) What is the probability that at least one is red?

$$Pr(\text{at least one red}) = 1 - Pr(\text{none red})$$
$$= 1 - \left(\frac{220}{816}\right) = \frac{596}{816} \cong 0.7304$$ OK

c) What is the probability that all three are red?

Treating red balls as distinct, there are $\binom{6}{3}$ ways of choosing three distinct red balls from six.

$$Pr(\text{all red}) = \frac{\binom{6}{3}}{\binom{18}{3}} = \frac{20}{816} \cong 0.0245$$ OK

d) What is the probability that all three are red given that at least one is red?

$A \Rightarrow$ all three red
$B \Rightarrow$ at least one is red

$Pr(A \cap B) = Pr(A)$ since $A \subseteq B$

$Pr(A \mid B) = \dfrac{Pr(A \cap B)}{Pr(B)} = \dfrac{Pr(A)}{Pr(B)}$ $\longrightarrow$ from question c) / $\longrightarrow$ from qustion b)

$$Pr(A \mid B) = \frac{(20/816)}{(596/816)} = \frac{20}{596} \cong 0.0336$$

OR

e) What is the probability that all three are different colors given that at least two colors appear?

$A \Rightarrow$ All three different colors
$B \Rightarrow$ At least two colors appear

$A \cap B = A$, given that $A \subseteq B$

To calculate $Pr(B)$, think of defining set C where $C \Rightarrow$ all three balls are the same color.

Then $Pr(B) = 1 - Pr(C)$, since $C = B^c$.

Thus, $|C| = |\{\text{all white}\}| + |\{\text{all blue}\}| + |\{\text{all red}\}|$
$= \binom{6}{3} + \binom{6}{3} + \binom{6}{3}$, where balls are distinct.
$= 3 \cdot 20 = 60$

$$Pr(B) = 1 - Pr(C) = 1 - \left(\frac{60}{816}\right) = \frac{756}{816}$$

$|A| = |\{\text{ways to choose one white}\}| \cdot |\{\text{ways to choose one blue}\}| \cdot |\{\text{ways to choose one red}\}|$
$= \binom{6}{1}\binom{6}{1}\binom{6}{1} = 6^3 = 216$

$Pr(A) = 6^3/816 = \frac{216}{816}$

OR

$$\text{Hence, } Pr(A \mid B) = \frac{(6^3/816)}{(756/816)} = \frac{216}{756} \approx 0.2857$$