

2-13-(16)

a) Write out Pascal's triangle for $0 \leq n \leq 9$

16 A

17 A

	<u>K</u>									
<u>n</u>	0	1	2	3	4	5	6	7	8	9
0	1									
1	1	1								
2	1	2	1							
3	1	3	3	1						
4	1	4	6	4	1					
5	1	5	10	10	5	1				
6	1	6	15	20	15	6	1			
7	1	7	21	35	35	21	7	1		
8	1	8	28	56	70	56	28	8	1	
9	1	9	36	84	126	126	84	36	9	1

b) If a random number is chosen from the above triangle, what is the possibility that it is even?

$$A = \{ \text{even} \}$$

$$S = \{ \text{all entries} \}$$

$$|S| = \frac{(10)(10+1)}{2} = 55$$

$$P(A) = \frac{|A|}{|S|} = \frac{22}{55} = 0.4$$

$$|A| = 22$$

c) Write out $(a+b)^9$

$$(a+b)^9 = \sum_{k=0}^9 \binom{9}{k} a^k b^{9-k}$$

$$= a^9 + 9a^8b + 36a^7b^2 + 84a^6b^3 + 126a^5b^4 + 126a^4b^5 + 84a^3b^6 + 36a^2b^7 + 9ab^8 + b^9$$

2-13-(17)

a) What is coefficient of x^2y^5 when $(2x-y)^7$ is completely expanded?

$$\text{Let } u=2x, v=-y \Rightarrow (2x-y)^7 = (u+v)^7. \quad \text{Coefficient of } u^2v^5 \Rightarrow \binom{7}{2}$$

$$u^2v^5 = (2x)^2(-y)^5 = (2^2)(-1)^5 x^2 y^5 \Rightarrow \text{Coefficient} = \binom{7}{2} (2^2)(-1)^5 = \frac{7!}{(7-2)!2!} (2^2)(-1)^5 = \frac{7 \cdot 6 \cdot 5!}{5! \cdot 2!} \cdot 4 \cdot (-1) = -84$$

b) What is coefficient of $u^{10}v^{12}$ when $(u^2+v^3)^9$ is fully expanded?

$$\text{Let } x=u^2, y=v^3 \Rightarrow (x+y)^9 = \sum_{k=0}^9 \binom{9}{k} x^k y^{9-k} \Rightarrow x^{10}y^{12} = (u^2)^5 (v^3)^4 \Rightarrow \begin{matrix} k=5 \\ n-k=4 \Rightarrow n=9 \end{matrix}$$

$$\text{Coefficient is } \binom{9}{k} = \binom{9}{5} = \frac{9!}{(9-5)!5!} = \frac{9!}{4!5!} = 126$$