Andrew Maule
2/10/2012
M331
Homework

2-10- 14 a) How many ways can 75 distinguishable books be put onto 5 distinct shelves?

14A
15A

From class, treat as dividers instead of shelves:

- (5-1) dividers for 5 shelves, so # of ways to arrange 75 books + 4 dividers where order matters:
- 1^{st} treat dividers as distinct:
  $(75+4)! = 79!$ ways to arrange
- 2^{nd} treat dividers as indistinguishable and divide by 4! ways in which they are overcounted

- Answer is: $\frac{79!}{4!}$

b) What is the probability that there are exactly two books on the 4^{th} shelf?

- First choose 2 books from 75 and put on 4^{th} shelf: # ways = $P(75, 2)$ where order matters
- Then take remaining 73 books and figure out # of ways to put on remaining 4 shelves. This should be the same as putting 73 distinguishable books on 4 shelves
  $= \frac{(73+3)!}{3!} = \frac{76!}{3!}$
- Then, for each arrangement on 4 shelves, insert shelf w/ 2 books b/w 3^{rd} & 4^{th} shelf to make 5 shelves → answer is product of two
- $= P(75, 2) \cdot \frac{76!}{3!} = \frac{75!}{73!} \cdot \frac{76!}{3!} = \frac{76! \cdot 75 \cdot 74}{3!}$

↳ over

- given equal probabilities of any arrangement:

$$Pr(\text{2 books on } 4^{th} \text{ shelf}) = \frac{\#\text{ arrangements w/ 2 books on shelf 4}}{\#\text{ arrangements of 75 books on 5 shelves}}$$

$$= \frac{\left(\frac{76! \cdot 75 \cdot 74}{3!}\right)}{\left(\frac{79!}{4!}\right)}$$

$$= \frac{\cancel{76!} \cdot 75 \cdot 74 \cdot 4 \cdot \cancel{3!}}{79 \cdot 78 \cdot 77 \cdot \cancel{76!} \cdot \cancel{3!}}$$

$$= (75 \cdot 74 \cdot 4)/(79 \cdot 78 \cdot 77)$$

$$\cong 0.0468$$

2-10 15 Assume a seven-letter word w/ letters A-Z where each letter is equally likely to be chosen.

a) What is the probability at least one letter in the word occurs at least twice?

- 1^{st} enumerate all possible 7-letter words where letters can be repeated:

$= 26^7$

- 2^{nd}; Pr(@ least one letter occurs @ least twice) = 1 - Pr(no letter appears twice)
  - ↳ number of ways no letter can appear twice is the same as $P(26, 7)$
  - ↳ Thus, Pr(@ least one letter occurs @ least twice) =

$$1 - \left[P(26,7)/26^7\right] = 1 - \left[\frac{\cancel{26} \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20}{26^{\cancel{7}\,6}}\right]$$

$$= 1 - \frac{25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20}{26^6} \boxed{\cong 0.587}$$

b) Pr(A does not appear in word)?

- count number of ways A does not appear, which is 25 choices per position (repeats allowed), or 25^7

- Thus, $Pr(\text{A does not occur}) = \frac{25^7}{26^7} \boxed{= 0.76}$

↳ In class, the question was Pr(A occurs @ least twice) = 1 - [Pr(A doesn't occur) + Pr(A occurs once)]

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(Continued)

2-10-15 b) → continued

$= 1 - [Pr(A \text{ doesn't occur}) + Pr(A \text{ occurs once})]$

$= 1 - \left[\frac{25^7}{26^7} + \frac{7 \cdot P(25,6)}{26^7} \right]$

7 places to put an A and P(25, 6) ways to arrange remaining 25 letters in 6 positions

$\cong 0.129$

c) Pr(@ least one vowel (AEIOU) occurs in word) =

1 - Pr(no vowel appears in word)

$= \cancel{1 - \frac{P(21,7)}{26^7}}$ ~~ways to arrange~~

$= 1 - \frac{21^7}{26^7} = 1 - \left(\frac{21}{26}\right)^7 \boxed{\cong 0.776}$

ways to select 21 remaining nonvowels in 7-letter word