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Math 331
1/30/2012

1 A
2 A
3 A
4 A

① Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

(i) $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

$x \in A \cup (B \cap C) \Rightarrow x \in A$ or $x \in B \cap C$

$\Rightarrow x \in A$ or $\{x \in B$ and $x \in C\}$

$\Rightarrow \{x \in A$ or $x \in B\}$ and $\{x \in A$ or $x \in C\}$

$\Rightarrow x \in A \cup B$ and $x \in A \cup C$

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$

$\therefore A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

(ii) $A \cup (B \cap C) \supseteq (A \cup B) \cap (A \cup C)$

$x \in (A \cup B) \cap (A \cup C) \Rightarrow x \in A \cup B$ and $x \in A \cup C$

$\Rightarrow \{x \in A$ or $x \in B\}$ and $\{x \in A$ or $x \in C\}$

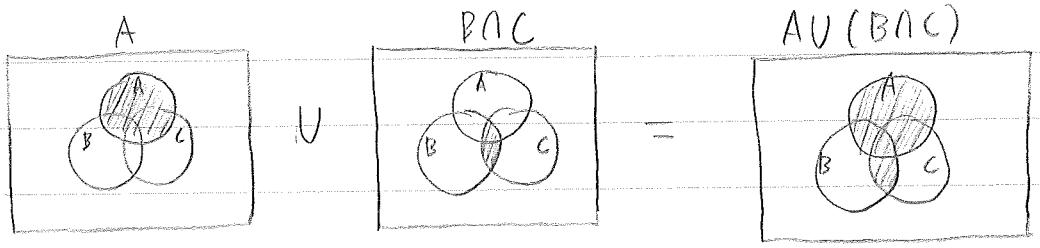
$\Rightarrow x \in A$ or $\{x \in B$ and $x \in C\}$

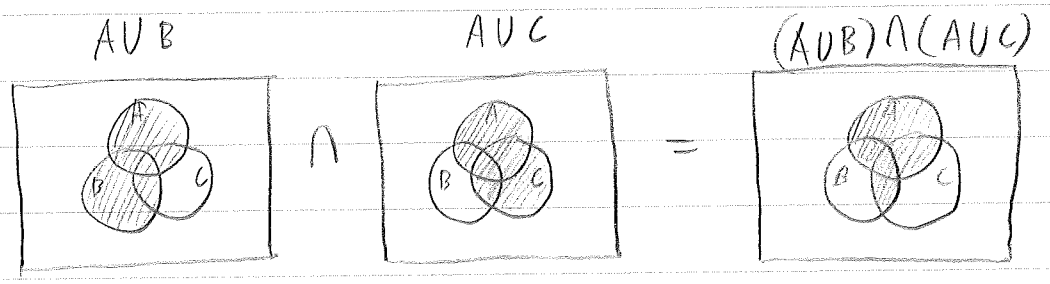
$\Rightarrow x \in A$ or $x \in B \cap C$

$\Rightarrow x \in A \cup (B \cap C)$

$\therefore (A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$

By (i) and (ii), $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

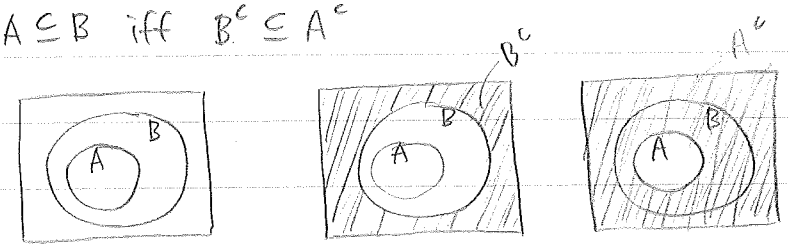




② Prove that $A \setminus B = B^c \setminus A^c$ and that $A \subseteq B$ iff $B^c \subseteq A^c$

(i) $A \setminus B = A \cap B^c = B^c \cap A = B^c \cap (A^c)^c = B^c \setminus A^c$

(ii) $A \subseteq B$ iff $B^c \subseteq A^c$



$A \subseteq B \iff (x \in A \Rightarrow x \in B)$

$\iff (x \notin B \Rightarrow x \notin A)$

$\iff B^c \subseteq A^c$

③ Prove that $(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$.

$(A \setminus B) \cup (B \setminus A) = (A \cap B^c) \cup (B \cap A^c)$

$= \{(A \cap B^c) \cup B\} \cap \{(A \cap B^c) \cup A^c\}$

$= \{(A \cup B) \cap (B^c \cup B)\} \cap \{(A \cup A^c) \cap (B^c \cup A^c)\}$

$A \cup A^c = S$ → $= \{(A \cup B) \cap S\} \cap \{S \cap (B^c \cup A^c)\}$

$= (A \cup B) \cap (B^c \cup A^c)$

$= (A \cup B) \cap (B \cap A)^c$

$= (A \cup B) \setminus (B \cap A)$

$= (A \cup B) \setminus (A \cap B)$

④ Prove that $(A \Delta B) \Delta C = A \Delta (B \Delta C)$.

* $A \Delta B = (A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$