

No calculators.
No notes.
Show all work.
Circle your answer.
Hand in to your TA.

Name_____

Circle the time of your TA section:

8:50-Laghi 9:55-Laghi 11:00-Basamakov 12:05-Basamakov 1:20-Jenkins 2:25-Jenkins

Problem	Points	Score
1	15	
2	15	
3	15	
4	15	
5	15	
6	15	
7	15	
8	15	
9	15	
10	15	
Total	150	

1. (15 pts) Find the derivative of $f(t) = \log_2(t^4 - t^2 + 1)$.

2. (15 pts) The half-life of Carbon-14 is 5730 years. A prehistoric human bone is analyzed and found to contain only 25 percent of the Carbon-14 which it had when the creature walked the earth. How many years ago did this person live?

3. (15 pts) Suppose g is the inverse function of a differentiable function f and let $h(x) = (g(x))^2$. Suppose $f(3) = 2$, $f(2) = 1$, $f'(2) = 3$, and $f'(3) = \frac{1}{9}$. Find $h'(2)$.

4. (15 pts) In certain engineering groups it is customary to let 90 degrees be referred to as 100 engineering degrees. So, for example, 25 engineering degrees is the same as 25% of 90 degrees, or 22.5 degrees. The function $\text{engsin}(x)$ is the ordinary sine function using engineering degrees instead of radians. Thus, for example, $\text{engsin}(50) = \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$. Find the derivative of $\text{engsin}(x)$ with respect to x .

5. (15 pts) Find the following limit:

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{k}{n}\right)^6 \left(\frac{1}{n}\right)$$

6. (15 pts) Find the definite integral:

$$\int_0^1 x \cos(x^2) \, dx$$

7. (15 pts) Sketch the region bounded by the curves $x = 1 - y^2$ and $x = y^2 - 1$ and find the area.

8. (15 pts) Graph the region bounded by the curve

$$x^2 + (y - 3)^2 = 4$$

This region is rotated about the x -axis. Find the volume of the resulting solid.

9. (15 pts) Define the function f by

$$f(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1 \\ x^2 + 1 & \text{if } 1 < x \leq 2 \end{cases}$$

Sketch the graph of f . Find the volume of the solid obtained by rotating the region under the graph of f about the x -axis.

10. (15 pts) Sketch the region in the plane bounded by

$$y = x^2$$

$$y = 0$$

$$x = 1$$

$$x = 2$$

Use the method of cylindrical shells to find the volume of the solid obtained by rotating this region about the line $x = 1$.

Answers

1.

$$\frac{4t^3 - 2t}{(t^4 - t^2 + 1) \ln(2)}$$

2. $2 \times 5730 = 11460$ years ago.

3. 54

4. $\frac{\pi}{200} \operatorname{eng} \cos(x)$ 5. $\int_0^1 x^6 dx = 1/7$ 6. $1/2 \sin(1)$. Substitute $u = x^2$.7. $4 \int_0^1 (1 - y^2) dy = \frac{8}{3}$

8. The region is a disk of radius $r = 2$ centered at the point $(0, 3)$. The solid is a torus. The volume of a torus is $(2\pi R)(\pi r^2)$ where a disk of radius r is rotated about a point R units away. In this case $R = 3$ and $r = 2$ so the volume is $24\pi^2$.

9. $\int_0^1 \pi(2x)^2 dx + \int_1^2 \pi(x^2 + 1)^2 dx = \frac{198}{15}\pi$ 10. $\int_1^2 2\pi(x - 1)x^2 dx = \frac{17}{6}\pi$