

Exam 1

A. Miller

Spring 99

Math 272

No calculators.
 Show all work.
 Circle your answer.
 Hand in to your TA.

Name_____

Circle the time of your TA section:

8:50-Laghi 9:55-Laghi 11:00-Basamakov 12:05-Basamakov 1:20-Jenkins 2:25-Jenkins

Problem	Points	Score
1	4	
2	4	
3	4	
4	4	
5	4	
6	4	
7	4	
8	4	
9	4	
10	6	
11	6	
12	6	
13	6	
14	8	
15	8	
16	8	
17	8	
18	8	
Total	100	

1. (4 pts) Find the equation for the parabola that has its vertex at the origin and its focus at $(0, -5)$.

2. (4 pts) Sketch the graph of the equation

$$4(x - 2)^2 + y^2 = 16$$

3. (4 pts) Classify the following according to whether they are a parabola, ellipse, or hyperbola, by drawing a connecting line.

$$12x^2 + (y - 3)^2 = 144$$

$$xy = 1$$

parabola

$$x + 2y^2 - y + 3 = 0$$

$$x^2 - y^2 = 1$$

ellipse

$$12x^2 - (y + 3)^2 = 144$$

$$x^2 + y^2 = 1$$

hyperbola

$$y = x^2$$

$$4x - 2x^2 - y^2 + 17 = 0$$

4. (4 pts) Suppose $f(x) = Ca^x$ is an exponential function such that the graph of f meets the y -axis at the point $(0, 3)$ and contains the point $(2, 12)$. What is $f(x)$?

5. (4 pts) Find the inverse function of $f(x) = \frac{x-2}{x+3}$.

6. (4 pts) Suppose you invest 1234 dollars at 6 percent yearly interest compounded continuously. After 40 days how much will you have? (You need not simplify your answer.)

7. (4 pts) Use the Laws of Logarithms to expand the expression below as much as possible.

$$\ln \left(\sqrt{\frac{x^e}{(1-x^2)}} \right)$$

8. (4 pts) Rewrite the expression below as a single logarithm.

$$3 \log(xy) - 2 \log(x - y) + 5 \log(x + y)$$

9. (4 pts) Suppose

$$f(x) = \frac{1}{(1 + \ln(x))^2}$$

What is $f'(e)$?

10. (6 pts) Find all solutions of the equation

$$\log(x^2 + 1) = 1 + 2\log(x)$$

11. (6 pts) The half-life of Carbon-14 is 5730 years. A prehistoric human bone is analyzed and found to contain only 12.5 percent of the Carbon-14 which it had when the creature walked the earth. How many years ago did this person live?

12. (6 pts) Find $\lim_{x \rightarrow 0} \left(\frac{1}{\ln(1+x)} - \frac{1}{x} \right)$.

13. (6 pts) Find $\lim_{x \rightarrow 1} x^{\frac{4}{x-1}}$.

14. (8 pts) What is the domain of the function

$$\log_{\log(x)}(x)$$

15. (8 pts) Suppose g is the inverse function of a differentiable function f and let $h(x) = \frac{1}{g(x)}$. Suppose $f(3) = 2$, $f(2) = 1$, $f'(2) = 3$, and $f'(3) = \frac{1}{9}$. Find $h'(2)$.

16. (8 pts) On a hot day a thermometer is taken outside from an airconditioned room. The room temperature is 70 degrees. After one minute the thermometer reads 80 degrees and after two minutes it reads 85 degrees. What is the temperature outside?

17. (8 pts) The function $y = e^x$ satisfies the differential equation

$$\frac{dy}{dx} = y$$

So does any constant multiple of it, $y = Ce^x$ for some constant C .

Explain why, if y is any solution of this differential equation, then there exists a constant C such that $y = Ce^x$. (Hint: I did this twice in lecture.)

18. (8 pts) State L'Hospital's rule for computing

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$$

and explain why it is true in the special case that $g'(a) \neq 0$ and the indeterminate form is $\frac{0}{0}$.

Answers

1. $-20y = x^2$
2. This an ellipse thru the points $(0, 0), (4, 0), (2, 4), (2, -4)$.
3. ehphhepe
4. $f(x) = 3 \cdot 2^x$
5. $f^{-1}(y) = \frac{3y+2}{1-y}$
6. $1234e^{\frac{40}{365} \cdot 06}$
7. $(\frac{1}{2})e \ln(x) - (\frac{1}{2}) \ln(1-x) - (\frac{1}{2}) \ln(1+x)]$
8. $\log((xy)^3(x-y)^{-2}(x+y)^5)$
9. $-\frac{1}{4e}$
10. $x = \frac{1}{3}$. Note that $x = -\frac{1}{3}$ is not a solution.
11. 3×5730 years ago. After 5730 years there was 50% left, after another 5730 years there was 25% left, and after the third 5730 there was 12.5% left.
12. $\frac{1}{2}$
13. e^4
14. All $x > 1$. $\log_a(x)$ is only defined for $a > 0$ and $a = \log(x) > 0$ only when $x > 1$.
15. It's -1 . $f(g(x)) = x$ so $f'(g(x))g'(x) = 1$ and so

$$g'(x) = \frac{1}{f'(g(x))}.$$

Since $f(3) = 2$ we know that $g(2) = 3$, hence

$$g'(2) = \frac{1}{f'(g(2))} = \frac{1}{f'(3)} = 9$$

and so by the quotient rule:

$$h'(x) = -\frac{1}{(g(x))^2} g'(x)$$

thus

$$h'(2) = -\frac{1}{(g(2))^2}g'(2) = -\frac{1}{9}9 = -1$$

16. 90 degrees. Let $T(t)$ be the temperature of the thermometer after t minutes. By Newton's Law of cooling

$$T(t) = A + Ce^{kt}$$

where A is the temperature outside and C and k are unknown constants. We are given

$$70 = T(0) = A + C$$

$$80 = T(1) = A + Ce^k = A + Cx$$

$$85 = T(2) = A + Ce^{k^2} = A + Cx^2$$

where we have let the unknown e^k be written x . Subtracting the first equation from the second and the third for the second gives

$$10 = Cx - C = C(x - 1)$$

$$5 = Cx^2 - Cx = x[C(x - 1)]$$

Substituting 10 for $C(x - 1)$ into the second equation gives $5 = x10$ and so $x = 1/2$, $C = -20$, and $A = 90$.

17. Suppose $\frac{dy}{dx} = y$. Let $u = ye^{-x}$. By the product rule

$$\frac{du}{dx} = \frac{dy}{dx}e^{-x} + ye^{-x}(-1) = ye^{-x} - ye^{-x} = 0$$

This means the derivative of u is always zero, hence u is a constant function. So there exists a constant C such that $u \equiv C$ and so $ye^{-x} = u = C$ and hence $y = Ce^x$.

18. See pages 420-421. Note 3 on page 421 was done twice in lecture.