No notes, no books, no calculators, no cell phones, no pagers, no electronic devices of any kind.

Name______________________________________

Circle your Discussion Section:

DIS 301	9:55	T	B305 VAN VLECK
DIS 302	9:55	R	115 INGRAHAM
DIS 305	1:20p	T	B105 VAN VLECK
DIS 306	1:20p	R	B333 VAN VLECK

Problem	Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	12	
9	4	
Total	100	

Solutions will be posted shortly after the exam:
www.math.wisc.edu/∼miller/m240

1. (12 pts) Give the best Big-Oh estimate for each of these functions. Enter one of the letters in the blank provided. (Answers may be reused.)

1. ___ $(2\log(n)+1)(3n+1)$

2. ___ $2n\log(n)+n^2$

3. ___ $n\log(n^2+n+1)+3n^2\log(n)+2n^2$

4. ___ $3\log(n)+5n+7$

(A) $\mathcal{O}(n)$

(B) $\mathcal{O}(n\log(n))$

(C) $\mathcal{O}(n\log^2(n))$

(D) $\mathcal{O}(n^2)$

(E) $\mathcal{O}(n^2\log(n))$.

(F) none of these

2. (12 pts)

(a) Determine which amounts of postage can be formed using just 3-cent and 10-cent stamps. (b) Prove your answer is correct using either simple induction or strong induction. Be sure to explicitly state your inductive hypothesis in the inductive step.

3. (12 pts) A parking lot has 30 spaces numbered $1, 2, \ldots, 30$ which are assigned by the hashing function

$$h(k) = (k \bmod 30) + 1$$

where k is the integer corresponding to the first 3 digits on the car's licence plate. Enter the correct parking space number $1, 2, \ldots, 30$ in each blank:

(a) Car with license plate starting 123 would park in space _____ .

(b) Car with license plate starting 234 would park in space _____ .

(c) Car with license plate starting 432 would park in space _____ .

4. (12 pts) Show that for positive integers a and b that

$$ab = gcd(a, b) \cdot lcm(a, b)$$

Hint: Use the prime factorization of a and b and the formulas for $gcd(a, b)$ and $lcm(a, b)$ in terms of these factorizations.

5. (12 pts) Perform the indicated base conversions:

(a) $(\underline{\hspace{3cm}})_8 = (1010111)_2$

(b) $(\underline{\hspace{3cm}})_8 = (\text{E2F})_{16}$

(c) $(\underline{\hspace{3cm}})_2 = (53)_{10}$

(d) $(\underline{\hspace{3cm}})_{10} = (37)_8$

6. (12 pts) Using the Euclidean algorithm to express the greatest common divisor d of $n = 123$ and $m = 1280$ as a linear combination of n and m:

$$d = \alpha(123) + \beta(1280) \text{ where } \alpha = \_\_\_\_\_\_ \text{ and } \beta = \_\_\_\_\_\_.$$

7. (12 pts) Find a matrix A such that

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$

Hint: Finding A requires that you solve systems of linear equations.

8. (12 pts) Prove that for every positive integer n:

$$1 \cdot 2 \ + \ 2 \cdot 3 \ + \ \cdots \ + \ n(n+1) = \frac{n(n+1)(n+2)}{3}$$

9. (4 pts) True or False.

(a) Matrix multiplication satisfies the commutative law.

(b) Matrix multiplication satisfies the associative law.

(c) If A and B are 2×2 matrices and

$$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

then

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ or } B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

(d) For any n there is an $n \times n$ matrix I such that for every $n \times n$ matrix A

$$AI = IA = A.$$

Answers

1. BDEA

2. (a) 3,6,9,10,12,13,15,16, and all $n \geq 18$.

(b) Verify directly that it is true for $n = 18, 19, 20$. Prove by strong induction that it is true for every $n > 20$. Since if $n > 20$ then $n - 3 \geq 18$. Hence by inductive hypothesis $n-3 = \alpha 3 + \beta 10$ for some nonnegative integers α and β. But then $n = (\alpha + 1)3 + \beta 10$.

3. (a) 4 (b) 25 (c) 13

4. see p.S-21 3.5-27.

For another proof: Let $d = gcd(a, b)$ and $a = a'\ d$ and $b = b'\ d$ where a' and b' are relatively prime. Put $m = a'\ d\ b'$. Then clearly $ab = dm$ and m is a common multiple of a and b. So it is enough to see that m is the least common multiple. Suppose n is any common multiple of a and b and let $n = \alpha a = \beta b$. Then since $\alpha\ a'\ d = \beta\ b'\ d$ we have that $\alpha\ a' = \beta\ b'$. Since a' and b' are relatively prime we have that a' divides β. So let $\beta = \gamma\ a'$. But then $n = \beta b = (\gamma a')(db')$ and so $m = a'\ d\ b'$ divides n.

5. (a) 127 (b) 7057 (c) 110101 (d) 31

6. $(32)(1280) + (-333)(123) = 1$

7.

$$A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}.$$

8. see 4.1-15.

9. FTFT