Put your answers on the answer sheet.
Do not hand in this exam.
You may separate these sheets and use the backs for scratch paper.

1. (Please read this problem carefully.) To maximize a real valued function $f(x, y)$ find all points (x, y) in the domain of f where

(A) $f_{xx}f_{yy} - (f_{xy})^2 > 0$ and $f_{xx} > 0$

(B) $f_{xx}f_{yy} - (f_{xy})^2 > 0$ and $f_{xx} < 0$

(C) $f_{xx}f_{yy} - (f_{xy})^2 < 0$ and $f_{xx} < 0$

(D) $f_{xx}f_{yy} - (f_{xy})^2 < 0$ and $f_{xx} > 0$

(E) None of above

2. The gradient of f points in the direction

(A) of smallest increase of f

(B) of greatest decrease of f

(C) of greatest increase of f

(D) in which the directional derivative is zero.

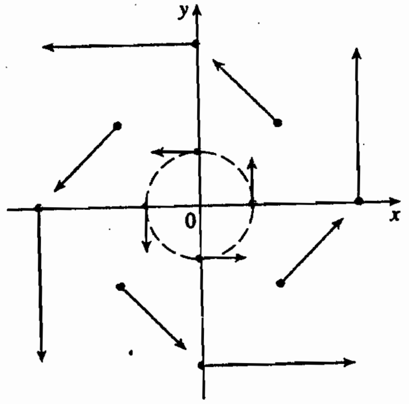
(E) None of above

3. The vector field $\mathbf{F}$ is sketched in the figure. Let C_1 be the unit circle (dotted) traversed in a counterclockwise fashion. The line integral $\int_{C_1} \mathbf{F} \odot \mathbf{dr}$ is...

(A) positive

(B) negative

(C) zero



4. For the same vector field, let C_2 be the line segment starting at $(-2, 2)$ and ending at $(2, 2)$. The line integral $\int_{C_2} \mathbf{F} \odot \mathbf{dr}$ is...

(A) positive

(B) negative

(C) zero

Match the parameterization with its surface.

5.
$x = s\cos t$
$y = s\sin t$
$z = s^2$

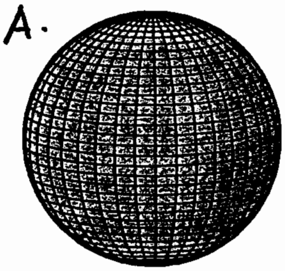
6.
$x = s\cos t$
$y = s\sin t$
$z = s$

7.
$x = \sin s\cos t$
$y = \sin s\sin t$
$z = \cos s$

8.
$x = (3+\cos s)\cos t$
$y = (3+\cos s)\sin t$
$z = \sin s$

9.
$x = s\cos t$
$y = s\sin t$
$z = t$

A.



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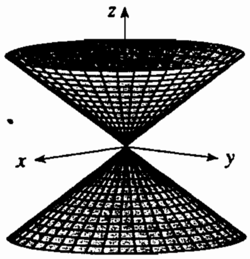
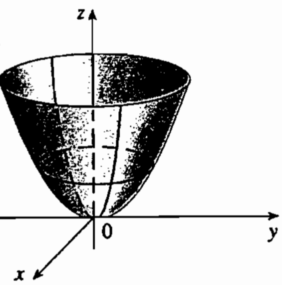
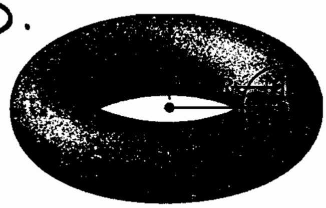


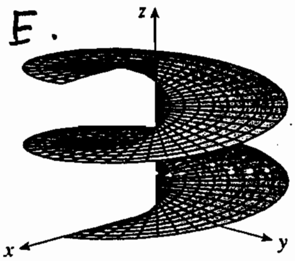
C.



D.



E.



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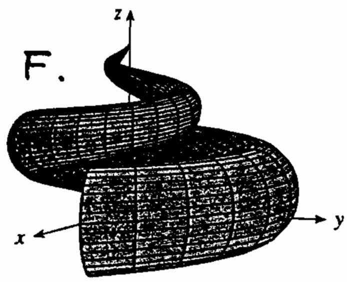


G. None of these

Match the vector field with its equation

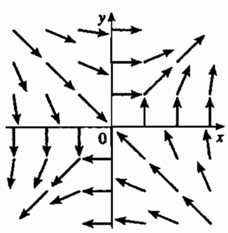
10. $\mathbf{F}(x,y) = x\,\mathbf{i} + y\,\mathbf{j}$

11. $\mathbf{F}(x,y) = \frac{y}{\sqrt{x^2+y^2}}\,\mathbf{i} + \frac{-x}{\sqrt{x^2+y^2}}\,\mathbf{j}$

12. $\mathbf{F}(x,y) = \frac{y}{\sqrt{x^2+y^2}}\,\mathbf{i} + \frac{x}{\sqrt{x^2+y^2}}\,\mathbf{j}$

13. $\mathbf{F}(x,y) = y\,\mathbf{i} + \mathbf{j}$

A.



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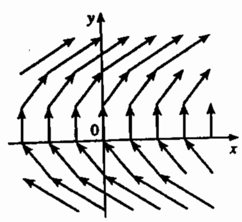

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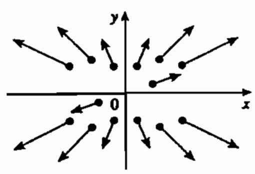


D. None of these

14. For any point on the level surface $g(x, y, z) = C$ the gradient of g will be

(A) in the tangent plane to the surface pointing in the direction in which g is increasing

(B) pointing toward the origin

(C) in the tangent plane to the surface pointing in the direction in which g is decreasing

(D) orthogonal to the tangent plane

(E) None of above

Match the function with its contour graph

15. $f(x, y) = x^2 + 9y^2$

16. $f(x, y) = \sqrt{x + y}$

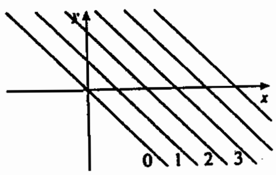
17. $f(x, y) = \frac{x}{y}$

18. $f(x, y) = xy$

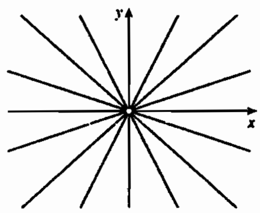
19. $f(x, y) = x^2 - y^2$

20. $f(x, y) = x^2 - y$

A.



D.



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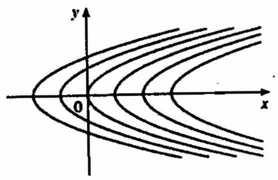

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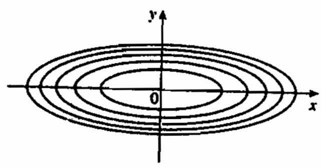

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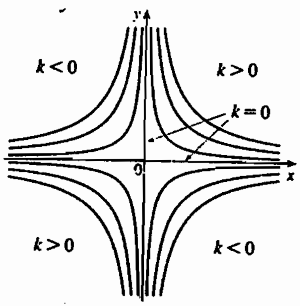


F. None of these

21. $\mathbf{F}(x,y) = 3x^2y^2\,\mathbf{i} \;+\; 2xy^3\,\mathbf{j}$ is a

(A) conservative vector field

(B) nonconservative vector field

(C) None of above

22. The vector field $\mathbf{F}(x,y) = (2xy^3 + 2y + 1)\,\mathbf{i} \;+\; (3x^2y^2 + 2x)\,\mathbf{j}$ is the gradient field of the function

(A) $f(x,y) = 2y^3 + 6xy$

(B) $f(x,y) = xy^3 + 2yx + x + x^2y^3 + x^2$

(C) $f(x,y) = x^2y^3 + 2xy + x$

(D) $f(x,y) = x^2 + xy^3$

(E) f exists but is none of above

(F) No f exists

Fill-in the Blank.

$$\int_0^2 \int_0^{x^2} f(x,y)\,dy\,dx = \int_{24.......}^{23.......} \int_{26.......}^{25.......} f(x,y)\,dx\,dy$$

(A) $\sqrt{y}$

(B) 0

(C) $\sqrt{x}$

(D) 4

(E) x^2

(F) 2

(G) $\sqrt{4-y}$

(H) $\sqrt{2-y}$

(I) None of above

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}} (x^2+y^2)^2 \, dy \, dx = \int_{28.......}^{27.......} \int_{30.......}^{29.......} 31....... \, dr \, d\theta$$

(A) 0

(B) r

(C) r^5

(D) r^3

(E) π

(F) 4

(G) 2

(H) $\frac{\pi}{2}$

(I) $-\pi$

(J) None of above

The figure shows the region E of integration.

$$\int\int\int_E g(x,y,z)dV = \int_0^1 \int_{33.......}^{32.......} \int_{35.......}^{34.......} g(x,y,z) \, dz \, dy \, dx$$

(A) 0

(B) y^2

(C) 1

(D) $\sqrt{x}$

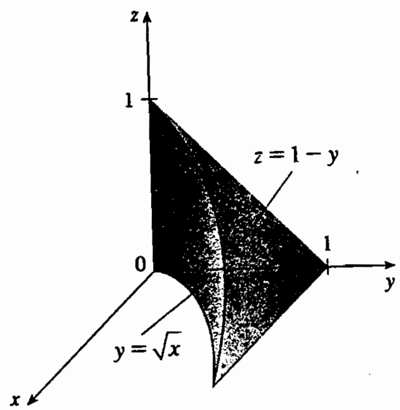
(E) $1-z$

(F) $1-y$

(G) $1-\sqrt{x}$

(H) $(1-z)^2$

(I) None of above



Show all work on the **answer sheet**, not here.

36. Find

$$\iint_S y\, dS$$

where S is the surface of the helicoid (or spiral ramp). It is parameterized by the equations:

$x = r\cos(\theta)$
$y = r\sin(\theta)$
$z = \theta$
$0 \le r \le 1, \quad 0 \le \theta \le \pi$

37. Find the point on the surface $z^2 = 2y - 2x + 4$ that is closest to the origin.

38. A particle starts at the point $(-1,0)$, moves along the x-axis to $(1,0)$, and then along the semicircle $y = \sqrt{1-x^2}$ back to the starting point. Call this curve C. Find

$$\oint_C (x^2e^x - y + xy^2)dx \quad + \quad (x + e^y\cos(y) + x^2y)dy$$

Hint: What does Green's Theorem say?

Take this exam home with you.

Scoring: Problems 1-35 one point each. Problems 36-38 five points each.

Answers will be posted on the web tonite:
www.math.wisc.edu/~miller/m234/index.html
