

No calculator, no cell phones, no pagers, no electronic devices at all.

You may bring in a single “cheat sheet”. One ordinary 8.5 x 11 sheet of paper with anything you want written on both sides.

Name_____

Hand in to your TA – Circle your TA section:

DIS 321	T 8:50	Milind Shah
DIS 322	R 8:50	Milind Shah
DIS 323	T 9:55	Milind Shah
DIS 324	R 9:55	Milind Shah
DIS 325	T 11:00	Duygu Unlu
DIS 326	R 11:00	Duygu Unlu
DIS 327	T 12:05p	Duygu Unlu
DIS 328	R 12:05p	Duygu Unlu
DIS 329	T 1:20p	Kiran Manchikanti
DIS 330	R 1:20p	Kiran Manchikanti
DIS 331	T 2:25p	Lipika Dekka
DIS 332	R 2:25p	Lipika Dekka
DIS 333	T 3:30p	Lipika Dekka
DIS 334	R 3:30p	Lipika Dekka

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. (10 pts) Find the domain of the vector function:

$$\mathbf{r}(t) = \ln(9 - t^2) \mathbf{i} + \sqrt{1+t} \mathbf{j} + \sin(t) \mathbf{k}$$

2. (10 pts) For what value of t is the normal plane to the curve

$$\mathbf{r}(t) = (t^3, t^2, 5t)$$

parallel to the plane $6x + 4y + 10z = 5$?

3. (10 pts) Find the **velocity, acceleration, and speed** of a particle with the position function:

$$\mathbf{r}(t) = \cos(t) \mathbf{i} + \sin(t) \mathbf{j} + t \mathbf{k}$$

Sketch the path of the particle for $t \geq 0$. Include in your sketch (at any point of your choice) the tangent $\mathbf{T}$, normal $\mathbf{N}$, and binormal $\mathbf{B}$.

4. (10 pts) Find and sketch the domain of the function:

$$f(x, y) = \ln(y - x^2)$$

5. (10 pts) Determine whether the function below is continuous at $(0, 0)$ and prove it:

$$f(x, y) = \begin{cases} \frac{xy}{x^2+2y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

6. (10 pts) Find all the **first** and **second** order partial derivatives of the function

$$f(x, y) = Ax^2 + Bxy + Cy^2 + Dx + Ey + F$$

7. (10 pts) Show that the function $f(x, y) = x^2y$ is (Frechet) differentiable at every point (x, y) by finding the values ϵ_1 and ϵ_2 where $\epsilon_1\Delta x + \epsilon_2\Delta y$ is the error term in the definition of differentiable.

8. (10 pts) The height y and width x of a rectangle change with time. At a certain instant the dimensions are $x = 30$ and $y = 40$ feet. At that same time x is increasing at the rate of 4 feet per second, and y is increasing at the rate of 2 feet per second. Find the rate at which the diagonal of the rectangle is increasing.

9. (10 pts) Find the maximum rate of change of $f(x, y) = ye^x + y$ at $(0, 1)$.

10. (10 pts) Find the point (or points) on the surface $z^2 = y - 2x + 1$ that is closest to the origin.

Answers

1. $D = \{t : -1 \leq t < 3\}$

2. $t = 1$. $\mathbf{r}'(1) = (3, 2, 5)$ which is parallel to $(6, 4, 10)$.

3. This is a helix see page 705. The normal vector and the acceleration vector are the same and point directly at the z -axis.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = -\sin(t) \mathbf{i} + \cos(t) \mathbf{j} + \mathbf{k}$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = -\cos(t) \mathbf{i} + -\sin(t) \mathbf{j}$$

4. $D = \{(x, y) : y > x^2\}$. This is the set of points in the plane strictly above the parabola.

5. It is discontinuous at $(0, 0)$. Along the line $x = 0$ the limit is 0 but along the line $y = x$ the limit is $1/3$.

6.

$$f_x = 2Ax + By + D$$

$$f_{xx} = 2A$$

$$f_y = Bx + 2Cy + E$$

$$f_{yy} = 2C$$

$$f_{xy} = f_{yx} = B$$

7. $\epsilon_1 \Delta x + \epsilon_2 \Delta y = (\Delta x)^2 y + 2x \Delta x \Delta y + (\Delta x)^2 \Delta y$

Many choices are possible, for example, $\epsilon_1 = (\Delta x)y + 2x \Delta y + \Delta x \Delta y$ and $\epsilon_2 = 0$.

8. 4 feet per second

9. $\sqrt{5}$

10. The minimum of $g(x, y) = x^2 + y^2 + (y - 2x + 1)$ in the region

$$D = \{(x, y) : y - 2x + 1 \geq 0\}$$

occurs on the boundary of D , i.e. on the line $y - 2x + 1 = 0$. The critical point of g is not in the region D . The closest point is $x = 2/5$, $y = -1/5$ and $z = 0$.