

1. Find the equation of the parabola with directrix $y = -1$ and focus $F(0,0)$.
2. Given that the graph is nondegenerate, use the Discriminant Test to determine whether the graph is a parabola, ellipse, or hyperbola.

$$9x^2 + 6xy + 4y^2 + 6x - 22 = 0$$

3. Find the volume of the solid generated by rotating the curve $y = 1/x$, $1 \leq x < \infty$, about the x-axis. Show that the surface area is infinite.
4. Calculate the integral:

$$\int \frac{dx}{(1+x)\sqrt{x}}$$

5. Calculate the integral:

$$\int \sin(\theta) \cos(\theta) \cos(\sin(\theta)) d\theta$$

6. Calculate the integral:

$$\int \tan^6(\theta) d\theta$$

7. Calculate the integral:

$$\int \frac{x^3}{\sqrt{1+x^2}} dx$$

8. Find the area inside the spiral $r = \theta$, $0 \leq \theta \leq 2\pi$.
9. Find the surface area generated by rotating the curve

$$r = 1 + \cos(\theta), \quad 0 \leq \theta \leq \frac{\pi}{2}$$

about the x-axis.

10. Compute the standard part of

$$\frac{H + 1 + \epsilon}{2H - 1 + 3\epsilon}$$

where H is infinite and ϵ is infinitesimal.

1. Find all solutions of the equation:

$$\frac{dy}{dx} = e^{x-y}$$

2. Calculate the following limit:

$$\lim_{x \rightarrow 0^+} x^x$$

3. Find the derivative with respect to x of:

$$\log_x(2)$$

4. Find $\int \frac{1}{x^2+x^4} dx$.
5. Find $\int \sqrt{e^x + 1} dx$.
6. Find $\int \frac{1}{1+\sqrt{x}} dx$

7. Find $\int \frac{\log(x)}{(1+x)^2} dx$

8. For each of the following series fill in the blank with exactly one of the answers a,b,c,d or e which stand for:

- a) divergent
 - b) conditionally convergent
 - c) absolutely convergent
 - d) absolutely and conditionally convergent
 - e) none of above
- $\sum \frac{1}{n}$ is ____.
- $\sum \frac{(-1)^n}{n}$ is ____.
- $\sum \frac{(-1)^n}{n^2}$ is ____.
- $\sum \frac{(-1)^n}{\sqrt{n}}$ is ____.

9. Determine whether the following series converges or diverges, and state your reason:

$$\sum \frac{1}{n(\log(n))^2}$$

10. Find the radius of convergence and the interval of convergence:

$$\sum \frac{(-1)^n}{n2^n} x^{2n+1}$$

Suppose that $f(x) = \sum_{n=0}^{\infty} \frac{n+1}{3^n} x^n$.

- 11. Find the power series for $f'(x)$.
- 12. Find the power series for $\int_0^x f(t) dt$ where f(x) is given at the top of this page.
- 13. Find the polynomial

$$p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$$

which best approximates $\tan(x)$ for x's near zero.

14. Find a power series converging to $\sqrt{1-x^2}$.

Final Exam

A. Miller

Spring 87

Math 222

1. Find the volume of the solid generated by rotating the curve $y = 1/x$, $1 \leq x < \infty$, about the x-axis. Show that the surface area is infinite.

2. Find

$$\int \sqrt{1-x^2} dx$$

3. Use four terms of a Taylor series to approximate

$$\int_0^{\frac{1}{2}} \frac{dx}{1+x^3}$$

Give an estimate for the error. It is not necessary for you to simplify your answer by using a calculator.

4. Find a power series converging to $\sqrt{1-x^2}$.

5. What is the equation, in the form $ax+by+cz = d$, of the plane which contains the three vectors: $\mathbf{i}+\mathbf{j}+2\mathbf{k}$, $2\mathbf{i}+\mathbf{j}+\mathbf{k}$, and $\mathbf{i}+\mathbf{k}$?

6. Find the position vector $\mathbf{X}(t)$ of a particle with the velocity vector

$$\mathbf{V}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$$

and initial position (time $t=0$)

$$\mathbf{X}(0) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$$

Where will the particle be at time $t=1$?

What is the speed the particle is moving at time $t=2$?

What is the acceleration vector at time $t=3$?

7. Find the angle between the vectors: $3\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{i} - \mathbf{j}$. Find a unit vector which is orthogonal to both of these vectors.

8. Find the vector equation of the line containing the vectors: $\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{i} - \mathbf{j}$.

Practice Final Exam

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Math 222

1. Find

$$\int \sqrt{1+x^2} dx$$

2. Evaluate the limit

$$\lim_{x \rightarrow \infty} \frac{(\ln x)^2}{\sqrt{x}}$$

3. Find all solutions of the differential equation

$$\frac{dy}{dx} = \sqrt{xy}$$

4. Use four terms of a Taylor series to approximate

$$\int_0^{\frac{1}{2}} \frac{dx}{1+x^3}$$

Give an estimate for the error. It is not necessary for you to simplify your answer by using a calculator.

5. What is the equation, in the form $ax + by + cz = d$, of the plane which contains the three points: $(1, 0, 1)$, $(-2, 1, -3)$, and $(1, 2, 3)$?

6. Let

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{(n!)^2}$$

What is the radius of convergence? What is the power series of the function $f^{(2)}(x)$? What is the power series (up to a constant) of $\int f(x) dx$? What is $f^{(48)}(0)$?

7. Find $\frac{dy}{dx}$ if $y = x^{\ln(x)}$.

8. Find

$$\lim_{n \rightarrow \infty} n \sqrt{\frac{1}{n}}$$

9. Find

$$\int \frac{2x^3 + 4x + 2}{x^4 - 1} dx$$

10. Calculate the series:

$$\frac{2 \times 3}{4} + \frac{2 \times 3 \times 3}{4 \times 4} + \frac{2 \times 3 \times 3 \times 3}{4 \times 4 \times 4} + \dots$$

11. Find

$$\int \frac{\sqrt{x} dx}{1 + \sqrt[4]{x}}$$

12. Define sequence. Define what it means for a sequence to converge. Define what it means for a sequence to diverge. What is the difference between a sequence and a series? (Your answer to this question should be about four or five sentences.)

13. Are the following series absolutely convergent, conditionally convergent, or divergent? Say why your answer is correct.

$$a) \sum_{n=1}^{\infty} \frac{(-1)^n}{n(n+1)}$$

$$b) \sum_{n=2}^{\infty} \frac{(-1)^n}{n} \left(\frac{1}{\log(n)} \right)^{\frac{3}{2}}$$

14. Starting with the geometric series show how to obtain the Taylor series of $f(x) = \arctan(x^3)$. What is $f^{(47)}(0)$?

15. Find the intervals of convergence for the following series:

$$a) \sum_{n=0}^{\infty} \frac{n}{2^n 3^{n+1}} x^{2n}$$

$$b) \sum_{n=0}^{\infty} n^2 (x+1)^n$$

16. Let $\mathbf{A} = i - 2j + k$ and $\mathbf{B} = -i + j$. Find the following:

- a) $\mathbf{A} + 4\mathbf{B}$
- b) $\mathbf{A} \cdot \mathbf{B}$
- c) $\|\mathbf{A}\|$
- d) the line containing $\mathbf{A}$ and $\mathbf{B}$
- e) $\mathbf{A} \times \mathbf{B}$
- f) the plane containing $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{0}$
- g) the point on the line in d) closest to $\mathbf{0}$

17. Let $\mathbf{S} = 2 \cos(2\pi t)i + 3 \sin(2\pi t)j + tk$. Find the velocity, speed, and acceleration.