

1. Prove that $\lim_{\theta \rightarrow 0} \frac{\sin(\theta)}{\theta} = 1$.
2. Find $\int_0^\infty \frac{x}{(1+x^2)^2} dx$ or show it diverges.
3. Find $\int \arctan(\frac{1}{x}) dx$.
4. Find $\int \sec^2(\theta) \sin^3(\theta) d\theta$.
5. Find $\int \frac{\cos(x) \sin^3(x)}{\sqrt{1+\sin^2(x)}} dx$.
6. Find the area between the curves (given in polar coordinates) $r = \sin(\theta)$ and $r = \cos(\theta)$ for $0 \leq \theta \leq \frac{\pi}{4}$.
7. Find the length of the curve $r = \sec(\theta)$ for $0 \leq \theta \leq \frac{\pi}{4}$.

1. Prove that

$$1 + b + b^2 + \dots + b^{n-1} = \frac{1 - b^n}{1 - b}.$$

2. Find $\frac{dy}{dx}$ if $y = x^{\ln(x)}$.

3. Find

$$\lim_{n \rightarrow \infty} n^{\sqrt{\frac{1}{n}}}$$

4. Find

$$\int \frac{2x^3 + 4x + 2}{x^4 - 1} dx$$

5. Calculate the series:

$$\frac{2 \times 3}{4} + \frac{2 \times 3 \times 3}{4 \times 4} + \frac{2 \times 3 \times 3 \times 3}{4 \times 4 \times 4} + \dots$$

6. Find

$$\int \frac{\sqrt{x} dx}{1 + \sqrt[4]{x}}$$

7. Define sequence. Define what it means for a sequence to converge. Define what it means for a sequence to diverge. What is the difference between a sequence and a series? (Your answer to this question should be about four or five sentences.)

1. Are the following series absolutely convergent, conditionally convergent, or divergent? Say why your answer is correct.

$$\begin{aligned} a) & \sum_{n=1}^{\infty} \frac{(-1)^n}{n(n+1)} \\ b) & \sum_{n=2}^{\infty} \frac{(-1)^n}{n} \left(\frac{1}{\log(n)} \right)^{\frac{3}{2}} \end{aligned}$$

2. Let $f(x) = f(0) + f^{(1)}(0)x + E(x)$ and $0 < x$. Show that there exists ζ with $0 < \zeta < x$ such that

$$E(x) = \frac{f^{(2)}(\zeta)}{2!}x^2.$$

3. Starting with the geometric series show how to obtain the Taylor series of $f(x) = \arctan(x^3)$. What is $f^{(47)}(0)$?

4. Find the intervals of convergence for the following series:

$$a) \sum_{n=0}^{\infty} \frac{n}{2n3^{n+1}} x^{2n}$$

$$b) \sum_{n=0}^{\infty} n^2(x+1)^n$$

5. Let $\mathbf{A} = i - 2j + k$ and $\mathbf{B} = -i + j$. Find the following:

- a) $\mathbf{A} + 4\mathbf{B}$
- b) $\mathbf{A} \cdot \mathbf{B}$
- c) $\|\mathbf{A}\|$
- d) the line containing $\mathbf{A}$ and $\mathbf{B}$
- e) $\mathbf{A} \times \mathbf{B}$
- f) the plane containing $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{0}$
- g) the point on the line in d) closest to $\mathbf{0}$

Final Exam

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Fall 85

Math 222

1. Find

$$\int \sqrt{1+x^2} dx$$

2. Evaluate the limit

$$\lim_{x \rightarrow \infty} \frac{(\ln x)^2}{\sqrt{x}}$$

3. Find all solutions of the differential equation

$$\frac{dy}{dx} = \sqrt{xy}$$

4. Use four terms of a Taylor series to approximate

$$\int_0^{\frac{1}{2}} \frac{dx}{1+x^3}$$

Give an estimate for the error. It is not necessary for you to simplify your answer by using a calculator.

5. What is the equation, in the form $ax + by + cz = d$, of the plane which contains the three points: $(1, 0, 1)$, $(-2, 1, -3)$, and $(1, 2, 3)$?

6. Let

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{(n!)^2}$$

What is the radius of convergence? What is the power series of the function $f^{(2)}(x)$? What is the power series (up to a constant) of $\int f(x) dx$? What is $f^{(48)}(0)$?

7. A 40 foot ladder is propped up against a 15 foot wall. What angle should the ladder make with the ground if the horizontal distance the ladder extends beyond the wall is to be a maximum?

8. Prove that if $\mathbf{X} \times \frac{d\mathbf{X}}{dt} = \beta \mathbf{k}$ then $r^2 \frac{d\theta}{dt} = \beta$, where β is a scalar constant and $\mathbf{X} = x\mathbf{i} + y\mathbf{j}$ and $x = r \cos(\theta)$, $y = r \sin(\theta)$.