501. (Variations of Parameters) Given the equation

$$\mathfrak{L}(z) = z'' + a_0 z' + a_1 z \equiv b$$

where a_0, a_1, b are given functions of t. Then

$$\mathfrak{L}(fz_1 + gz_2) = b$$

where

$$z_h = C_1 z_1 + C_2 z_2$$

is the general solution of the associated homogenous equation $\mathfrak{L}(z) \equiv 0$ and the derivatives of f and g satisfy:

$$f' = \frac{\det\begin{pmatrix} 0 & z_2 \\ b & z_2' \end{pmatrix}}{\det\begin{pmatrix} z_1 & z_2 \\ z_1' & z_2' \end{pmatrix}} \qquad g' = \frac{\det\begin{pmatrix} z_1 & 0 \\ z_1' & b \end{pmatrix}}{\det\begin{pmatrix} z_1 & z_2 \\ z_1' & z_2' \end{pmatrix}}$$

Use these formulas to find the general solution of

$$z'' + z \equiv (\cos t)^2$$

502. Solve the initial value problem:

$$z'' + z \equiv (\tan t)^2$$

$$z(0) = 1$$

$$z'(0) = -1$$

503. Find the general solution of

$$z'' - z \equiv \frac{1}{e^t + e^{-t}}$$

504. Given a system of linear equations

$$\begin{aligned} ax + by &= r \\ cx + dy &= s \end{aligned}$$

show that the solution is given by:

$$x = \frac{\det\begin{pmatrix} r & b \\ s & d \end{pmatrix}}{\det\begin{pmatrix} a & b \\ c & d \end{pmatrix}} \qquad y = \frac{\det\begin{pmatrix} a & r \\ c & s \end{pmatrix}}{\det\begin{pmatrix} a & b \\ c & d \end{pmatrix}}$$

You may assume the determinant:

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

is not zero.

Hint: Multiply the first equation by d and the second by $-b$ then add them.

505. Given the linear operator $\mathcal{L}(z) = z'' + a_0 z' + a_1 z$ suppose $\mathcal{L}(z_1) \equiv 0$ and $\mathcal{L}(z_2) \equiv 0$ and that f and g are functions of t which satisfy $f' z_1 + g' z_2 \equiv 0$. Show that

$$\mathcal{L}(f z_1 + g z_2) = f' z_1' + g' z_2'$$