Instructions

No notes, no books, no calculators, no cell phones, no pagers, no electronic devices of any kind.

Show all of your work. Circle your answer.

Name____________________________________

Circle your section number.
Hand in to your TA.

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Section
Number
 321  7:45  Zhao, Jie
 332  9:55  McMahon, Kayla
 322  8:50  Zhao, Jie
 328  1:20  Emrah, Elnur
 331  9:55  Kim, Yoosik
 324 11:00  McMahon, Kayla
 325 12:05  Wang, Kejia
 326 12:05  Emrah, Elnur
 327  1:20  Wang, Kejia
 330  2:25  Kim, Yoosik
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Problem	Points	Score
1	10	
2	8	
3	8	
4	8	
5	8	
6	8	
7	8	
8	8	
9	8	
10	8	
11	10	
12	8	
Total	100	

Solutions will be posted shortly after the exam:
www.math.wisc.edu/∼miller

1. (10 pts) Put your simplified answer in the box.

- For $z = 2 + 3i$ find:

  (a) $z^2 =$ $\boxed{}$

  (b) $\overline{z} =$ $\boxed{}$

  (c) $|z| =$ $\boxed{}$

  (d) $\frac{1}{z} =$ $\boxed{}$

- For $z = 2e^{3i}$ find:

  (a) $\arg(z) =$ $\boxed{}$

  (b) $|z| =$ $\boxed{}$

  (c) $z^2 =$ $\boxed{}$

  (d) $\frac{1}{z} =$ $\boxed{}$

- For $z = -\pi e^{\frac{\pi}{2}i}$ find:

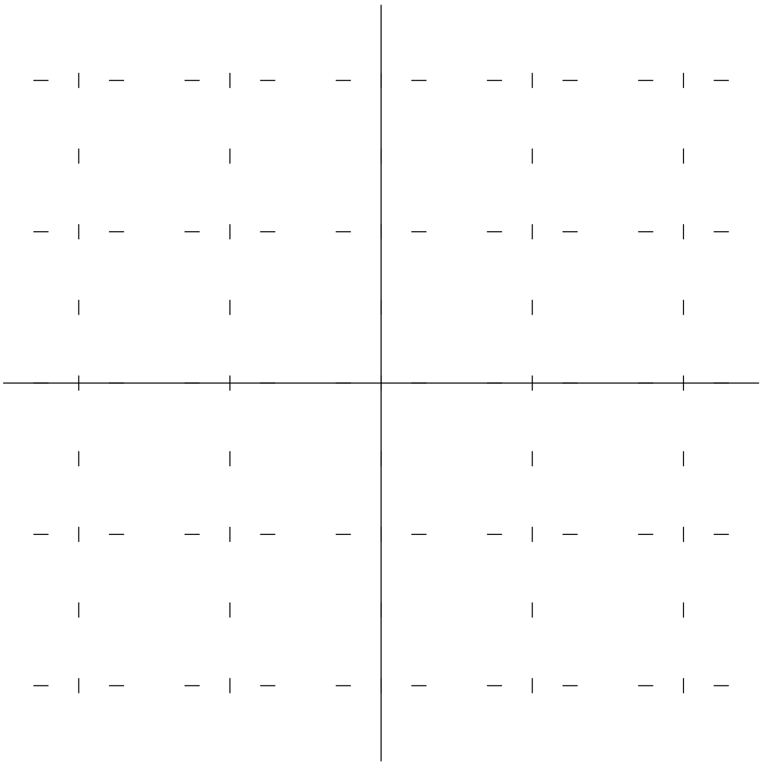
  (a) $|z| =$ $\boxed{}$

  (b) $\arg(z) =$ $\boxed{}$

2. (8 pts) Plot the following four points in the complex plane. Be sure and label them.

$P = \sqrt{2}\, e^{\frac{5\pi}{4} i}$ $\qquad Q = 1 + 2i$ $\qquad R = \overline{1 + 2i}$ $\qquad Z = \frac{1}{1+2i}$

The dotted lines are one unit apart.



3. (8 pts) Find all real or complex roots of the equation:

$$z^4 + z^2 - 12 = 0$$

Circle your answer.

4. (8 pts) Find the function y of x which satisfies the **initial** value problem:

$$\frac{dy}{dx} + \frac{x^2 - 1}{y} = 0 \qquad y(0) = 1$$

Circle your answer.

5. (8 pts) Find the **general** solution of

$$\frac{dy}{dx} + 2y + e^x \equiv 0$$

Circle your answer.

6. (8 pts) Solve the **initial** value problem:

$$y'' - 5y' + 4y \equiv 0$$

$$y(0) = 2$$
$$y'(0) = -1$$

Circle your answer.

7. (8 pts) For y as a function of x, find the **general** solution of the equation:

$$y'' - 2y' + 10y \equiv 0$$

Circle your answer.

8. (8 pts) Find a **particular** solution of the equation:

$$y'' + y' + 2y = e^x + x + 1$$

Circle your answer.

9. (8 pts) Given the equation $\mathcal{L}(z) = z'' + a_0 z' + a_1 z \equiv b$ where a_0, a_1, b are given functions of t. Then $\mathcal{L}(fz_1 + gz_2) = b$ where $z_H = C_1 z_1 + C_2 z_2$ is the general solution of the associated homogenous equation $\mathcal{L}(z) \equiv 0$ and the derivatives of f and g satisfy:

$$f' = \frac{\det\begin{pmatrix} 0 & z_2 \\ b & z_2' \end{pmatrix}}{\det\begin{pmatrix} z_1 & z_2 \\ z_1' & z_2' \end{pmatrix}} \qquad g' = \frac{\det\begin{pmatrix} z_1 & 0 \\ z_1' & b \end{pmatrix}}{\det\begin{pmatrix} z_1 & z_2 \\ z_1' & z_2' \end{pmatrix}}$$

Use these formulas to find the general solution of

$$z'' + z \equiv \frac{1}{\sin t}$$

Circle your answer.

10. (8 pts) According to *Newton's law of cooling* the rate $\frac{dT}{dt}$ at which an object cools is proportional to the difference $T - A$ between its temperature T and the ambient temperature A. The differential equation which expresses this is $\frac{dT}{dt} = k(T - A)$ where $k < 0$ and A are constants. Solve this equation and show that every solution satisfies $\lim_{t\to\infty} T = A$.

11. (10 pts)

$$\vec{a} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

Compute, simplify, and then circle each answer:

(a) $||\vec{a}||$

(b) $2\vec{a}$

(c) $||2\vec{a}||^2$

(d) $\vec{a} + \vec{b}$

(e) $3\vec{a} - \vec{b}$

12. (8 pts) Compute, simplify, and then circle each answer:

(a) Find a parametric equation for the line which contains the two vectors

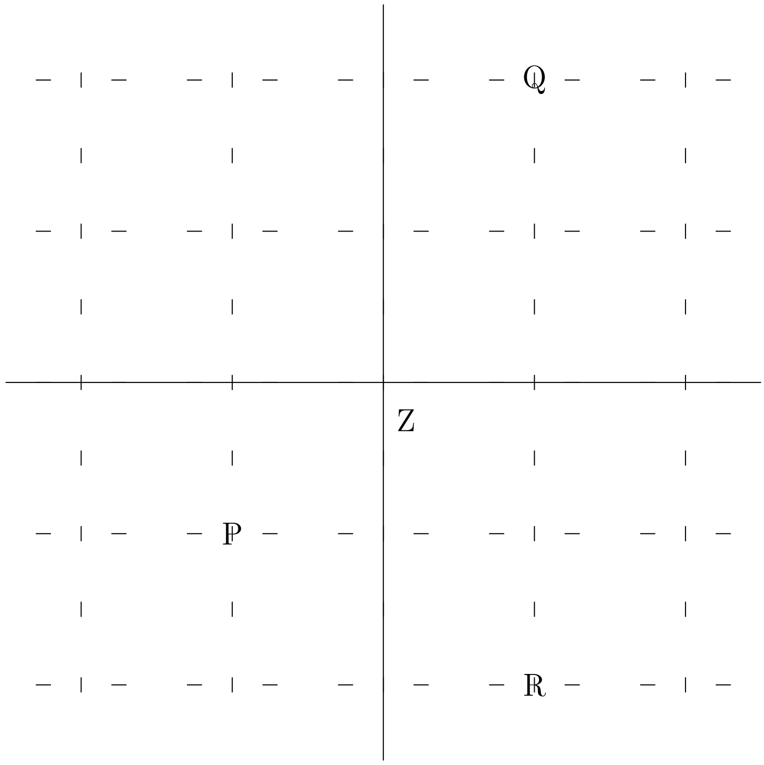
$$\vec{a} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \text{ and } \vec{b} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}.$$

(b) The vector $\vec{c} = \begin{pmatrix} c_1 \\ 1 \\ c_3 \end{pmatrix}$ is on this line. What is $\vec{c}$?

Answers

1. (a) $-5+12i$ (b) $2-3i$ (c) $\sqrt{13}$ (d) $\frac{2}{13}-\frac{3}{13}i$
(a) 3 (b) 2 (c) $4e^{6i}$ (d) $\frac{1}{2}e^{-3i}$
(a) π (b) $\frac{3}{2}\pi$

2.



3. $\sqrt{3}$, $2i$, $-\sqrt{3}$, $-2i$

4. $y=\sqrt{2(x-\frac{x^3}{3})+1}$

5. $y=Ce^{-2x}-\frac{1}{3}e^{x}$

6. $y=3e^{x}-e^{4x}$

7. $y=Ae^{x}\sin(3x+B)$

8. $y_P=\frac{1}{4}e^{x}+\frac{1}{2}x+\frac{1}{4}$

9. $z = (-t + C_1)\cos t \quad + \quad (\ln|\sin t| + C_2)\sin t$

10. $T = A + Ce^{kt}$
$\lim_{t\to\infty} e^{kt} = 0$

11.
(a) 3

(b) $\begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix}$

(c) 36

(d) $\begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix}$

(e) $\begin{pmatrix} 1 \\ -5 \\ 5 \end{pmatrix}$

12. (a) $\vec{L}[t] = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$
(b) $\begin{pmatrix} 4 \\ 1 \\ 5 \end{pmatrix}$