Instructions

No notes, no books, no calculators, no cell phones, no pagers, no electronic devices of any kind.

Show all of your work. Circle your answer.

Name____________________________________

Circle your section number.
Hand in to your TA.

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Section
Number
 321  7:45  Zhao, Jie
 332  9:55  McMahon, Kayla
 322  8:50  Zhao, Jie
 328  1:20  Emrah, Elnur
 331  9:55  Kim, Yoosik
 324 11:00  McMahon, Kayla
 325 12:05  Wang, Kejia
 326 12:05  Emrah, Elnur
 327  1:20  Wang, Kejia
 330  2:25  Kim, Yoosik
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Problem	Points	Score
1	9	
2	9	
3	9	
4	9	
5	9	
6	9	
7	9	
8	9	
9	7	
10	6	
11	9	
12	6	
Total	100	

Solutions will be posted shortly after the exam:
www.math.wisc.edu/~miller

1. (9 pts) Show all of your work. Circle your answer.

Find

$$\int_{-1}^{2} |x^2 - x| \, dx$$

Express your answer as a simple fraction.

2. (9 pts) Show all of your work. Circle your answer.

Find

$$\int \frac{\ln(2x^2)}{x}\,dx$$

3. (9 pts) Show all of your work. Circle your answer.

Find

$$\int \frac{1}{3x^2 + 6x + 6}\, dx$$

4. (9 pts) Show all of your work. Circle your answer.

Find

$$\int \frac{x^3}{x^2 - 1}\, dx$$

5. (9 pts) Show all of your work. Circle your answer.

Find

$$\int \frac{1}{x^2(x+1)}\,dx$$

6. (9 pts) Show all of your work. Circle your answer.

Find
$$\int 2x \ln(x+1)\, dx$$

7. (9 pts) Show all of your work. Circle your answer.

Find a second order polynomial (i.e., a quadratic function) $p(x)$ such that $p(2) = 3$, $p'(2) = 8$, and $p''(2) = -1$.

8. (9 pts) Show all of your work. Circle your answer.

Find the Taylor series for the following function, by substituting adding, multiplying, applying long division,and/or differentiating known series, for $\sin(x), \cos(x), e^x, \ln(1+x)$, or $\frac{1}{1-x}$.

$$f(x) = \begin{cases} \frac{\sin(x)}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

9. (7 pts) Show all of your work. Circle your answer.

(a) Find the second degree Taylor polynomial for the function e^t.
(b) Use it to give an estimate for the integral

$$\int_0^1 e^{x^2}\, dx$$

Express your answer as a simple fraction.

10. (6 pts) Show all of your work. Circle your answer.

Suppose instead we used the 5th degree Taylor polynomial $p(t)$ for e^t to give an estimate for the integral:

$$\int_0^1 e^{x^2}\, dx$$

Give an upper bound for the error:

$$\left| \int_0^1 e^{x^2}\, dx - \int_0^1 p(x^2)\, dx \right|$$

Note: You need not find $p(t)$ or the integral $\int_0^1 p(x^2)\, dx$ and will be given no credit for doing so. You do not need to simplify your answer. Yes, one million is an upper bound, but you will get no credit for it.

11. (9 pts) Show all of your work. Circle your answer.

For which real numbers x does the Taylor series of $f(x) = \frac{1}{2-5x}$ converge to $f(x)$?

12. (6 pts) Circle T or F for True or False. No penalty for guessing.

T F Given a function f and distinct real numbers $a_1 < a_2 < \cdots < a_n$ there is a unique polynomial $p(x)$ of degree $\leq n-1$ such that $f(a_k) = p(a_k)$ for all $k = 1, 2, \ldots, n$.

T F Given any function f continuous on the interval $[a, b]$ and $\epsilon > 0$ there is a polynomial $p(x)$ such that $|f(x) - p(x)| < \epsilon$ for all x in $[a, b]$.

T F Given any f with n-derivatives at 0 there is a unique polynomial $p(x)$ of degree $\leq n$ such that $f^{(k)}(0) = p^{(k)}(0)$ for all $k = 0, \ldots, n$.

T F $\lim_{n\to\infty} a^n = 0$ if and only if $-1 < a < 1$.

T F $\lim_{n\to\infty} \frac{a^n}{n!} = 0$ for every a.

T F $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$

T F $\ln(2) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$

T F $\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$

T F If $f(x) = e^{-\frac{1}{x^2}}$ for $x \neq 0$ and $f(0) = 0$, then $f^{(n)}(0) = 0$ for all $n = 0, 1, 2, \ldots$

T F Since $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$, we have that

$$\frac{1}{2} = \frac{1}{1-(-1)} = 1 + (-1) + 1 + (-1) + \cdots$$

T F Now that we have studied advanced techniques of integration we know the correct thing we should have been writing all along is:

$$\frac{d}{dx}x^2 = 2x + C$$

T F Apply integration by parts to $\int \frac{1}{x}\,dx$ letting $u = \frac{1}{x}$ and $dv = dx$ gives

$$\int \frac{1}{x}\,dx = (\frac{1}{x})(x) - \int -(\frac{1}{x^2})x\,dx$$

simplifying gives us

$$\int \frac{1}{x}\,dx = 1 + \int \frac{1}{x}\,dx$$

and subtracting the integral from both sides gives us $0 = 1$.

Answers

1. $\frac{11}{6}$
2. $\frac{1}{4}(\ln(2x^2))^2 + C$
3. $\frac{1}{3}\arctan(x+1) + C$
4. $\frac{1}{2}(x^2 + \ln|x^2-1|) + C$
5. $-\ln|x| - \frac{1}{x} + \ln|x+1| + C$
6. $x^2\ln(x+1) - \frac{x^2}{2} + x - \ln(x+1) + C$
7. $p(x) = 3 + 8(x-2) - \frac{1}{2}(x-2)^2$
8. $f(x) = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}\, x^{2n}$
9. $\frac{43}{30}$
10. $\frac{3}{6!\cdot 13}$
11. $|x| < 2/5$.
12. The last three are false.