NAME_____ SCORE _____

Joel Robbin

# Calc 221 Final Exam, Monday December 15, 1997

*Circle your section.*

| | | | |
|---|---|---|---|
| 311 | Hollingsworth | 8:00 TR | B305 Van Vleck |
| 312 | Hollingsworth | 9:30 TR | B305 Van Vleck |
| 313 | Nilsen | 9:30 TR | B203 Van Vleck |
| 314 | Kent | 11:00 TR | B305 Van Vleck |
| 315 | Resnick | 11:00 TR | B309 Van Vleck |
| 316 | Kent | 1:00 TR | B305 Van Vleck |
| 317 | Resnick | 2:30 TR | B305 Van Vleck |
| 318 | Nilsen | 2:30 TR | 3335 Sterling |

| | | |
|---|---|---|
| I | 20 Points | |
| II | 20 Points | |
| III | 20 Points | |
| IV | 20 Points | |
| V | 20 Points | |
| VI | 20 Points | |
| VII | 20 Points | |
| VIII | 20 Points | |
| IX | 20 Points | |
| X | 20 Points | |
| Total | 200 Points | |
| Extra Credit | 25 Points | |

Show your reasoning.

NAME_____ SCORE _____

# Calc 221 Final Exam, Monday December 15, 1997

*Circle your section.*

| | | | |
|---|---|---|---|
| 331 | Herzig | 7:45 TR | B211 Van Vleck |
| 333 | Huedepohl | 9:55 TR | B329 Van Vleck |
| 334 | Huedepohl | 11:00 TR | B337 Van Vleck |
| 335 | Kung | 12:05 TR | B333 Van Vleck |
| 336 | Kung | 1:20 TR | B325 Van Vleck |
| 337 | Herzig | 2:25 TR | B321 Van Vleck |

| | | |
|---|---|---|
| I | 20 Points | |
| II | 20 Points | |
| III | 20 Points | |
| IV | 20 Points | |
| V | 20 Points | |
| VI | 20 Points | |
| VII | 20 Points | |
| VIII | 20 Points | |
| IX | 20 Points | |
| X | 20 Points | |
| Total | 200 Points | |
| Extra Credit | 25 Points | |

Show your reasoning.

NAME_____ SCORE _____

# Calc 221 Final Exam, Monday December 15, 1997

*Circle your section.*

| | | | |
|---|---|---|---|
| 001 | CAUGHMAN, JOHN | 8:50 | B223 Van Vleck |
| 002 | MILLER | 9:55 | B223 Van Vleck |
| 003 | KERSEY, SCOTT | 11:00 | B223 Van Vleck |
| 007 | LANG, MICHAEL | 12:05 | B223 Van Vleck |
| 010 | APPS, PHILIP | 1:20 | B223 Van Vleck |
| 011 | PONOMARENKO, VADIM | 2:25 | B223 Van Vleck |
| 015 | HILDEBRAND, JEFF | 3:30 | B223 Van Vleck |

| | | |
|---|---|---|
| I | 20 Points | |
| II | 20 Points | |
| III | 20 Points | |
| IV | 20 Points | |
| V | 20 Points | |
| VI | 20 Points | |
| VII | 20 Points | |
| VIII | 20 Points | |
| IX | 20 Points | |
| X | 20 Points | |
| Total | 200 Points | |
| Extra Credit | 25 Points | |

SHOW YOUR REASONING.

**I.** (20 points.) (a) Find $\dfrac{dy}{dx}$ when $y = \ln(2 + e^x)$.

(b) Find $\dfrac{d^2y}{dx^2}$ for $y$ as in (a).

**II.** (20 points.) (a) Evaluate $\displaystyle\int_2^3 \frac{x^2\,dx}{x^3-1}$.

(b) Evaluate $\displaystyle\lim_{x\to 3}\left(\frac{1}{x-3}\int_3^x \frac{\sin(t)}{t}\,dt\right)$.

**III.** (20 points.) A population of bacteria triples in three hours. Assuming exponential growth, how long does it take to double?

**IV.** (20 points.) Find the points on the hyperbola $y^2 - x^2 = 4$ which are closest to the point $(2, 0)$.

**V.** (20 points.) (a) Find the equation for the tangent line to the curve $y^2 = x^3 + 3$ at the point $(x, y) = (1, 2)$.

(b) Is this tangent line above the curve? Why or why not?

**VI.** (20 points.) (a) Evaluate $\lim_{n\to\infty} \sum_{i=1}^{n} \left[ 3\left(1 + \frac{2i}{n}\right)^5 - 6 \right] \frac{2}{n}$.

(b) Evaluate $\lim_{x\to\infty} \tan^{-1}(x)$.

**VII.** (20 points.) The velocity function for a particle moving along a line is

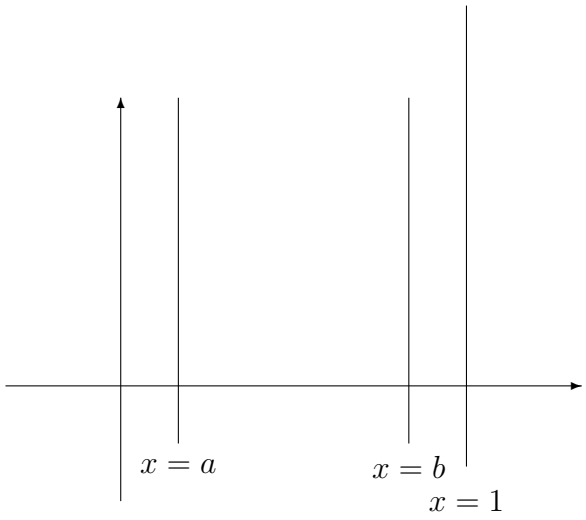
$$v(t) = 3t - 12, \qquad 0 \leq t \leq 5.$$

(a) Find the displacement (net change in position) from $t = 0$ to $t = 5$.

(b) Find the total distance travelled from $t = 0$ to $t = 5$. (Note that the velocity changes sign.)

**VIII.** (20 points.) Find the interval on which the curve $y = \int_0^x \frac{dt}{1+t+t^2}$ is concave up.

**IX.** (20 points.) Find the volume generated by revolving the region bounded by the lines $x = a$, $x = b$, $y = 0$ and the curve $y = \sqrt{1 - x^2}$ about the $x$-axis. (Assume that $a$ and $b$ are constants and $0 < a < b < 1$.)



**X.** (20 points.) State and prove the formula for the derivative of the inverse sine function $\sin^{-1}(x)$. You may assume without proof that the derivative of the sine function is the cosine function.

**EXTRA CREDIT.** (25 points.) A high speed train accelerates at (1/2) meter/sec$^2$ until it reaches its maximum cruising speed of 30 meters per second; after it reaches this maximum cruising speed it remains at that speed. If it starts from rest, how far will it go in 10 minutes?

Answers

Ia. $\frac{e^x}{2+e^x}$
Ib. $\frac{2e^x}{(2+e^x)^2}$
IIa. $\frac{1}{3}(\ln(26) - \ln(7))$
IIb. $\frac{\sin(3)}{3}$
III. $\frac{3\ln(2)}{\ln(3)}$
IV. $(1, \sqrt{5})$ and $(1, -\sqrt{5})$
Va. $(y-2) = \frac{3}{4}(x-1)$
Vb. Tangent line is below curve (at least locally) because concavity is upward.
VIa. $\frac{1}{2}3^6 - 12 - \frac{1}{2}$
VIb. $\frac{\pi}{2}$
VIIa. $-\frac{45}{2}$
VIIb. $\frac{51}{2}$
VIII. $(-\infty, -\frac{1}{2})$
IX. $\pi[(b - \frac{1}{3}b^3) - (a - \frac{1}{3}a^3)]$
EXTRA CREDIT 17100 meters