

final

A. Miller

Fall 95

Math 221

1. (6 %) Find a formula for the sum of the squares of the first n odd integers.
2. (6 %) Evaluate the integral $\int_0^{1/2} \frac{x^2}{\sqrt{1-x^6}} dx$.
3. (6 %) Find $\frac{dy}{dx}$ if $y = \int_1^{x^2} e^t \ln(t) dt$.
4. (6 %) Find the area bounded by the curves $y = x^3$ and $y = x^4$. (These two curves divide the whole plane into five pieces but only one of the pieces is bounded.)
5. (6 %) If the area bounded by the curves $y = x$, $y = x^2$, $x = 0$ and $x = 2$ is revolved around the x -axis, what is its volume?
6. (6 %) If the area bounded by the curves $y = x^2(x - 1)$, $y = 0$, $x = 0$ and $x = 1$ is revolved around the y -axis, what is its volume?
7. (8 %) Consider $\lim_{x \rightarrow 2} 3x = 6$. Given $\epsilon > 0$ find $\delta > 0$ so that $0 < |x - 2| < \delta$ implies $|3x - 6| < \epsilon$.
8. (8 %) The altitude of a triangle is increasing at the rate of 2 centimeter per second while the area of the triangle is increasing at rate 3 centimeters² per second. At what rate is the base of the triangle changing when the altitude is 5 centimeters and the area is 25 centimeters²?
9. (8 %) Find $\lim_{x \rightarrow \infty} \frac{\ln(1+e^{2x})}{3x}$
10. (8 %) Find the point on the line $y = 3 - 4x$ which is closest to the origin.
(2 points each) For the true-false problems on this and the next page, if the statement is true circle true. **If it is false, circle false and give an example (i.e., a specific $a, b, c, f, g, \dots$ or whatever) for which it is false.**
11. (True or False) If f is a differentiable function on an open interval containing $[a, b]$, then the integral $\int_a^b f(x)dx$ exists.
12. (True or False) If f is continuous on $[a, b]$ except possibly at a , then $\int_a^b f(x)dx$ exists.
13. (True or False) Areas cannot be negative.

14. (True or False) If f is Riemann integrable on the interval $[a, b]$, then

$$\int_a^b f(x)dx = \lim_{N \rightarrow \infty} \sum_{n=0}^{N-1} f(x_k) \Delta x$$

where $\Delta x = \frac{b-a}{N}$ and $x_0 = a$ and $x_{k+1} = x_k + \Delta x$.

In the problems below f and g are continuous functions on the given intervals. **Remember: If the statement is false, give an example (i.e., a specific $a, b, c, f, g, \dots$ or whatever) for which it is false.**

15. (True or False) $\int_a^b f(x)g(x)dx = \int_a^b f(x)dx \int_a^b g(x)dx$

16. (True or False) If $a < b$, then $\int_b^a f(x)dx \leq 0$.

17. (True or False) $\int_a^b [cf(x) + dg(x)]dx = c \int_a^b f(x)dx + d \int_a^b g(x)dx$ where c and d are constants.

18. (True or False) $\int_a^b |f(x)|dx = |\int_a^b f(x)dx|$.

19. (True or False) If $a < c < b$, then $\int_a^c f(x)dx \leq \int_a^b f(x)dx$.

20. (True or False) Suppose for every x in $[a, b]$, $f(x) \leq g(x)$, then

$$\int_a^b f(x)dx \leq \int_a^b g(x)dx.$$

21. (12 %) Prove part of the first part of the Fundamental Theorem of Calculus: Suppose $f(x)$ is a continuous function on the interval $[a, b]$ and we define

$$F(x) = \int_a^x f(t) dt,$$

then $F'(x) = f(x)$ for all x in (a, b) .

Answers

1. $4 \cdot \frac{n(n+1)(2n+1)}{6} - 4n(n+1)/2 + n$
2. $\frac{1}{3} \arcsin(\frac{6}{8})$
3. $(e^{x^2} \ln(x^2)) \cdot 2x$
4. $\frac{1}{20}$
5. 4π
6. $\frac{\pi}{20}$
7. $\delta = \epsilon/3$
8. $-\frac{14}{5}$
9. $\frac{2}{3}$
10. $(\frac{12}{17}, \frac{3}{17})$
11. true
12. false, $f(x) = \frac{1}{x}$ if $x \neq 0$ and $f(0) = 0$. This function is not Riemann integrable on $[0, 1]$ because it is unbounded.
13. true
14. true
15. false, $(\int_0^1 x dx) \cdot (\int_0^1 x dx) \neq \int_0^1 x^2 dx$
16. false, $\int_1^0 (-1) dx = 1$
17. true
18. false, $|\int_{-1}^1 x dx| = 0 \neq 1 = \int_{-1}^1 |x| dx$
19. false, $\int_1^2 (-1) dx = -1 > -2 = \int_1^3 (-1) dx$
20. true