

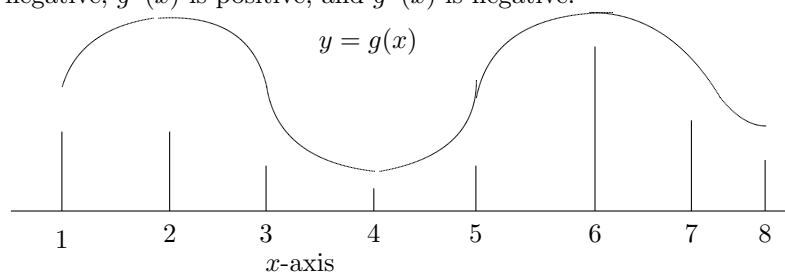
Exam 2

A. Miller

Fall 95

Math 221

1. (6 %) Find  $\frac{dy}{dx}$  if  $y = e^{x^e}$ .
2. (8 %) Byzantium decays at a rate proportional to the amount present. The half-life is 500 years. What fraction of the original amount is present after 250 years?
3. (6 %) Find  $\frac{dy}{dx}$  if  $y = \ln(x + \ln(x))$ .
4. (6 %) Find  $f(x)$  if  $f'(x) = x^4 + 3x^2 + 1$  and  $f(0) = 2$ .
5. (6 %) Let  $k(x)$  be the inverse of the function  $f(x) = x^5 + x$ . What is  $k'(2)$ ?
6. (6 %) Find  $\frac{dy}{dx}$  if  $y = \frac{1}{1 + \arctan^2(x)}$ .
7. (6 %) Find  $\lim_{x \rightarrow 0} (1 + 2x)^{\frac{3}{x}}$ .
8. (6 %) Find  $\lim_{x \rightarrow \infty} \frac{(\ln(x))^3}{x^2}$ .
9. (8 %) Two corridors, one 6 meters and the other 3 meters wide, meet at a right angle. A calculus instructor wants to wheel a flat portable blackboard around the corner. How long can the black board be?
10. (4 %) The function  $g(x)$  on the interval  $(1,8)$  has both a first and second derivative at every point. Find the intervals where  $g'(x)$  is positive,  $g'(x)$  is negative,  $g''(x)$  is positive, and  $g''(x)$  is negative.



11. (4 %) Draw the graph of a function  $f(x)$  such that  $f'(x)$  is positive on the intervals  $(1,2)$  and  $(3,4)$  and negative on the interval  $(2,3)$ .
12. (4 %) Draw the graph of function  $f(x)$  such that  $f''(x)$  is positive on the intervals  $(1,2)$  and  $(3,4)$  and negative on the interval  $(2,3)$ .

Multiple choice and True-False circle your answer. (1 point each)

13. Suppose  $f(x) = \theta$  is the inverse function of  $x = \sin(\theta)$  restricted to  $[\pi/2, \pi]$ . (Warning this is not the usual interval.) What is  $f'(x)$ ?
- $\arcsin(x)$
  - $\arcsin(x) + C$
  - $\frac{1}{\sqrt{1-x^2}}$
  - $\frac{-1}{\sqrt{1-x^2}}$
  - $\cos(x)$
  - none of above
14.  $\sin(\arcsin(\frac{7}{4}\pi)) = ?$
- $\frac{7}{4}\pi$
  - $-\frac{1}{4}\pi$
  - $\frac{1}{4}\pi$
  - undefined
  - none of above
15. The derivative of  $\log_x(x) = ?$
- 1
  - $x$
  - 0
  - undefined
  - none of above
16. If  $f$  is twice differentiable on  $(a, b)$ ,  $a < c < b$ ,  $f'(c) = 0$  and  $f''(c) > 0$  then
- $f$  has an inflection point at  $c$ .
  - $f$  has a local minimum at  $c$ .
  - $f$  has a local maximum at  $c$ .
  - $f$  may have a saddle point at  $c$ .
  - none of above.
17. (True or False) If  $f$  is a continuous function on  $[a, b]$  and differentiable on  $(a, b)$  and  $f'(a) = f'(b)$ , then  $f'(c) = 0$  for some  $c$  with  $a < c < b$ .
18. (True or False) If the derivative of a function is always positive then the function cannot have an inflection point.
19. (True or False) If  $f(x) = x^3 + x + 1$  then there must be a real number  $c$  with  $0 < c < 1$  such that  $f'(c) = 2$ .
20. (True or False) Suppose  $g'(x) > 0$  on an interval  $(a, b)$  then  $g(x)$  is a decreasing function.
21. (True or False) If an interval contains three distinct zeros of a twice differentiable function then it also contains a zero of the function's second derivative.

22. (True or False) A continuous function  $f$  on an interval  $(a, b)$  must achieve its extreme values (minimum and maximum).
23. (True or False) Between any two zeros of a differentiable function lies a zero of its derivative.
24. (True or False) If  $f(c)$  is the minimum value of a differentiable function  $f$ , then  $f'(c) < 0$ .
25. (True or False)  $\lim_{x \rightarrow 3} \frac{x-3}{x^2-3} = \lim_{x \rightarrow 3} \frac{1}{2x}$  by using L'Hôpital's Rule.
26. (True or False) Suppose  $g'(x) = 0$  for all  $x$ , then  $g$  is a constant function.
27. (True or False) If  $f'(x) = 2x$ , then  $f(x) = x^2$ .
28. (True or False) If  $f(x) = g(x)$  for all  $x$ , then  $f'(x) = g'(x)$  for all  $x$ .
29. (True or False) If  $f'(x) = g'(x)$  for all  $x$ , then  $f(x) = g(x)$  for all  $x$ .
30. (True or False) If  $f(x) \neq g(x)$  for all  $x$ , then  $f'(x) \neq g'(x)$  for all  $x$ .
31. (True or False) If  $f''(x)$  is positive on the interval  $(a, b)$ , then the function  $f'(x)$  is increasing on the interval  $(a, b)$ .
32. (True or False) There exist two nonconstant functions  $x$  and  $y$  of  $t$  such that  $\frac{dx}{dt} = y$  and  $\frac{dy}{dt} = x$ .
33. (10 %) Prove the mean value theorem. If a function  $f$  is continuous on  $[a, b]$  and differentiable on  $(a, b)$  then there exists a  $c$  with  $a < c < b$  and

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

You may use Rolle's Theorem without proof, but if so, you must correctly state it and check that it applies.

## Answers

1.  $e \cdot x^{e-1} \cdot e^{x^e}$
2. .707107
3.  $\frac{1}{x+\ln(x)} \cdot (1 + \frac{1}{x})$
4.  $f(x) = 2 + x + x^3 + \frac{x^5}{5}$
5.  $\frac{1}{6}$
6.  $\frac{-1}{(1+(\arctan(x))^2)^2} \cdot 2 \arctan(x) \cdot \frac{1}{1+x^2}$
7.  $e^6$
8. 0
9.  $\theta_0 = \arctan(\sqrt[3]{2})$  and  $B = \frac{6}{\sin(\theta_0)} + \frac{3}{\cos(\theta_0)}$
10.  $g'(x)$  is positive:  $(1, 2), (4, 6)$ ,  $g'(x)$  is negative:  $(2, 4), (6, 8)$ ,  $g''(x)$  is positive:  $(3, 5), (7, 8)$ , and  $g''(x)$  is negative:  $(1, 3), (5, 7)$ .
13. d
14. d
15. c
16. b
17. false
18. false
19. true
20. false
21. true
22. false
23. true
24. false
25. false
26. true
27. false
28. true
29. false
30. false
31. true
32. true