

Exam 1

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Fall 95

Math 221

1. If Newton's method is used to solve the equation $x^3 + x + 1 = 0$ with starting value $x_1 = -1$, what will be x_2 ?
2. Find the quadratic approximation to $f(x) = \frac{1}{x}$ near $x = 4$.
3. The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of $2 \text{ cm}^2/\text{min}$. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm^2 ?
4. Find a second-degree polynomial P such that $P(2) = 5$, $P'(2) = 3$ and $P''(2) = 2$.
5. Find the equation of the tangent line to the curve $y^2 = x^3(2 - x)$ at the point $(1, 1)$.
6. Find the derivative of the function $G(x) = (3x - 2)^{10}(5x^2 - x + 1)^{12}$.
7. Prove that $\frac{d}{dx}(\cot x) = -\csc^2 x$. You may use the usual rules of differentiation as well as the derivative of the sine and cosine without proving them.
8. Find the points on the curve $y = x^3 - x^2 - x + 1$ where the tangent line is horizontal.
9. Let $y = \sqrt{1 + x}$. Find $\frac{dy}{dx}$ directly from the definition of the derivative and not using any rules for taking derivatives.
10. Find $\lim_{x \rightarrow \infty} \frac{\sqrt{1+4x^2}}{4+x}$.
11. If $f(x) = x^3 - x^2 + x$, show that there is a number c such that $f(c) = 10$.
12. Prove that $\lim_{x \rightarrow 2} \frac{1}{x} = \frac{1}{2}$ using the $\epsilon - \delta$ definition of the limit.
13. Prove that $\lim_{x \rightarrow 0} x^2 \sin(\frac{1}{x}) = 0$. (Here you don't need to use $\epsilon - \delta$ unless you want to.)
14. Sketch the graph of the function

$$f(x) = \begin{cases} 1 - x & \text{if } x \leq 0 \\ 1 & \text{if } x > 0. \end{cases}$$

Use the graph to state the value of each of the following limits, if it exists.

1. $\lim_{x \rightarrow 0^-} f(x)$
2. $\lim_{x \rightarrow 0^+} f(x)$
3. $\lim_{x \rightarrow 0} f(x)$

15. Prove the product rule for differentiation:

$$\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}.$$

The following problems are multiple choice. Circle the best answer.

16. $\lim_{x \rightarrow 0} \frac{0}{x}$ is

1. undefined
2. $\frac{0}{0}$
3. 0
4. 1
5. none of above.

17. $\frac{d}{dx} 10^3 =$

1. $3 * 10^2$
2. 0
3. 1
4. 10^3
5. none of above

18. Every continuous function is

1. differentiable.
2. differentiable except at finitely many points.
3. not differentiable.
4. discontinuous.
5. none of above.

19. Every differentiable function

1. is continuous.
2. is continuous except possibly at finitely many points.
3. is a polynomial.
4. has the property that its graph cannot be drawn without lifting your pencil from the paper.

5. none of above.
20. If f is a real-valued function whose domain is every real number, then
1. it must be differentiable at least one point.
 2. it must be continuous at all except possibly finitely many points.
 3. it could be discontinuous at every point.
 4. it is a constant function.
 5. none of above.
21. Suppose $\lim_{x \rightarrow 1} f(x) = 0$ and $\lim_{x \rightarrow 1} g(x) = 0$, then the $\lim_{x \rightarrow 1} \frac{f(x)}{g(x)}$
1. is $\frac{0}{0}$.
 2. is $\frac{0}{0}$ which is equal to 1.
 3. is $\frac{0}{0}$ which is equal to 0.
 4. exists but could be anything.
 5. doesn't exist.
 6. either doesn't exist or it exists but it could be anything.
 7. none of above.

Answers

1. $x_2 = -\frac{3}{4}$
2. $p(x) = \frac{1}{4} - \frac{1}{16}(x-4) + \frac{1}{64}(x-4)^2$
3. $-\frac{8}{5} \text{ cm/sec}$
4. $P(x) = 3 - x + x^2$
5. $y = x$
6. $G'(x) = (10(3x-2)^9 \cdot 3)(5x^2-x+1)^{12} + (3x-2)^{10} \cdot 12 \cdot (5x^2-x+1)^{11} \cdot (10x-1)$
7. Use the quotient rule on $\cot(x) = \frac{\cos(x)}{\sin(x)}$
8. $(1, 0)$ and $(-\frac{1}{3}, \frac{32}{27})$
10. 2
11. Since $f(1) = 1$ and $f(3) = 21$ by the intermediate value theorem, there exists c with $1 < c < 3$ and $f(c) = 10$.
12. Choose $\delta > 0$ but less than the minimum of ϵ and 1.
13. Use the squeeze property and $-x^2 \leq x^2 \sin(1/x) \leq x^2$.
14. All three limits exist and equal 1.
16. 3
17. 2
18. 5
19. 1
20. 3
21. 6