

Exam 2

A. Miller

Fall 91

Math 221

- 1.(5%) Draw the graph of a function $f(x)$ such that $f'(x)$ is positive on the intervals $(1, 2)$ and $(3, 4)$ negative on the interval $(2, 3)$.
- 2.(5%) Draw the graph of function $f(x)$ such that $f''(x)$ is positive on the intervals $(1, 2)$ and $(3, 4)$ and negative on the interval $(2, 3)$.
- 3.(5%) Prove the special case of L'Hôpital's Rule: Suppose that $f(x)$ and $g(x)$ are differentiable at a , $f(a) = g(a) = 0$, and $g'(a) \neq 0$. Then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$.
- 4.(5%) Draw a picture illustrating the Mean Value Theorem.
- 5.(5%) A light is hung 12 feet directly above a straight horizontal walk on which a girl 5 feet tall is walking. How fast is the girl's shadow lengthening when she is walking away from the light at the rate of 168 feet per minute?
- 6.(5%) A woman has 320 yards of fencing for inclosing two separate lots, one of which is to square and other a rectangle three times as long as it is wide. Find the dimensions of each lot so that the total area inclosed shall be a minimum.
- 7.(5%) Prove the first part of the Fundamental Theorem of Calculus: Suppose $f(x)$ is a continuous function on the interval $[a, b]$ and defining $F(x) = \int_a^x f(t) dt$, then $F'(x) = f(x)$.
- 8.(5%) Prove the second part of the Fundamental Theorem of Calculus: Suppose $f(x)$ is a continuous function on the interval $[a, b]$ and $G(x)$ is a function such that $G'(x) = f(x)$, then $\int_a^b f(x) dx = G(b) - G(a)$.
- 9.(5%) Find $\frac{d}{dx} \int_0^{\sin(x)} \cos(t^4 + t^3) dt$.
- 10.(5%) Evaluate the definite integral $\int_0^1 \frac{x^3}{(x^4+1)^2} dx$.
- 11.(5%) Suppose $\int_2^3 3x^2 \tan(x^3) dx = \int_a^b \tan(u) du$. What are a and b ?
- 12.(5%) Suppose $\int_0^2 f(x) dx = 3$ and $\int_1^2 -4f(x) dx = 4$. What is $\int_0^1 f(x) dx$?
- 13.(5%) Solve $\frac{dy}{dx} = \left(\frac{x}{y}\right)^3$.
- 14.(5%) Solve $\frac{dy}{dx} = \sin^2(x)$.
- 15.(5%) Solve $\int x \tan^2(x^2 + 1) dx$.
- 16.(5%) Find $\lim_{x \rightarrow \pi/2} \frac{1 - \sin(x)}{1 - \cos(4x)}$.
- 17.(5%) Standing on a 160 foot tower you throw a rock upwards at the velocity of 64 feet per second. What is the maximum height that rock achieves? (Recall that the acceleration due to gravity is -32 feet per second².)

18.(5%) Use the Trapezoid rule with $n = 4$ to approximate the definite integral $\int_0^2 x^2 dx$.

For each of the following, circle True or circle False.

19.(1%) (True or False) $\lim_{x \rightarrow 3} \frac{x-3}{x^2-3} = \lim_{x \rightarrow 3} \frac{1}{2x}$ by using L'Hôpital's Rule.

20.(1%) (True or False) If $f(x)$ is a differentiable function, then it is integrable.

21.(1%) (True or False) If $f(x)$ is a continuous function, then it is integrable.

22.(1%) (True or False) If $f(x)$ is a function which is continuous on $[a, b]$, differentiable on (a, b) , and $f(a) = f(b)$, then for some c in (a, b) we have $f'(c) = 0$.

23.(1%) (True or False) $\int_1^0 f(x) dx = -\int_0^1 f(x) dx$ for any continuous function $f(x)$.

24.(1%) (True or False) $\int_1^0 f(x) dx < 0$ for any continuous function $f(x)$.

25.(1%) (True or False) If $f'(x) = g'(x)$ for all x , then $f(x) = g(x)$ for all x .

26.(1%) (True or False) If $f'(x) = 0$ for all x , then $f(x)$ is a constant function.

27.(1%) (True or False) If $f''(x)$ is positive on the interval (a, b) , then the function $f'(x)$ is increasing on the interval (a, b) .

28.(1%) (True or False) If $f'(x)$ is negative on the interval (a, b) , then the function $f(x)$ is increasing on the interval (a, b) .

Answers Exam 2 Fall 91

5. 288
6. $\frac{240}{7}$ by $\frac{160}{7}$
9. $\cos(x) \cdot \cos(\sin^4(x) + \sin^3(x))$
10. $\frac{1}{8}$
11. $a = 8$ and $b = 27$
12. 4
13. $\frac{y^4}{4} = \frac{x^4}{4} + C$
14. $y = \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$
15. $\frac{1}{2}(\tan(x^2 + 1) - (x^2 + 1)) + C$
16. $\frac{1}{16}$
17. 244
18. 2
19. False
20. True
21. True
22. True
23. True
24. False
25. False
26. True
27. True
28. False