

1. (2 %) Define what it means to say that $\lim_{x \rightarrow a} f(x) = L$.
2. (2 %) Explain what it means to say that a limit has the form $\frac{0}{0}$. Give an example of a limit of this form which turns out to be 2.
3. (2 %) Give an example of a limit which does not exist.
4. (2 %) Give an example where $\lim_{x \rightarrow a^+} f(x) = L_1$, $\lim_{x \rightarrow a^-} f(x) = L_2$, and $L_1 \neq L_2$.
5. (2 %) Define what it means to say that the function is continuous.
6. (2 %) Give an example of a function which is not continuous at the point 0.
7. (2 %) Define what it means to say that the function is differentiable.
8. (2 %) Give an example of a function which is continuous at every point but not differentiable at 0.
9. (2 %) Is it always the case that a differentiable function is continuous? Why or why not?
10. (2 %) Is it possible to have a function which is defined for every real number but is not continuous at any point? Why or why not?
11. (2 %) Suppose that a differentiable function has both negative values and positive values. Must it have zeros? Why or why not?
12. (2 %) Is it true that every function defined on an interval takes on a minimum value and a maximum value in that interval? If not, what must be true of that function and that interval?
13. (8 %) Give a direct proof without using any rules of differentiation that $\frac{d}{dx}x^2 = 2x$
14. (8 %) Show how to derive the quotient rule for differentiation:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v^2}$$

15. (8 %) Explain how to derive the formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ which is used in Newton's method to solve the equation $f(x) = 0$.
16. (5 %) $\lim_{x \rightarrow \infty} \frac{1-x^2}{1+4x+3x^2} =$
17. (4 %) $\lim_{x \rightarrow \infty} \frac{1}{2x^2} =$
18. (4 %) $\lim_{x \rightarrow 0^+} \frac{\sqrt{3x}}{x} =$

19. (4 %) $\frac{d}{dx}\pi^3 =$
20. (6 %) $\frac{d^2}{dx^2}(2x^4 - x^2 + x + 3) =$
21. (4 %) $\frac{d}{dx}(\sqrt[3]{x}) =$
22. (4 %) $\frac{d}{dx}(\sin^8(x)) =$
23. (4 %) $\frac{d}{dx}\left(\frac{\tan(x)}{x^3+1}\right) =$
24. (4 %) $\frac{d}{dx}(\sec(3x)\cos(x)) =$
25. (4 %) Suppose $x = t^2$ and $y = \sin(t)$. $\frac{dy}{dx} =$
26. (5 %) Suppose $y + x^2y^3 = 10$. Find $\frac{dy}{dx}$ when $x = 1$ and $y = 2$.
27. (4 %) If Newton's method is used to solve the equation $x^3 - 1 = x^2$ with starting value $x_1 = 1$, what will be x_3 ?

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2. $\lim_{x \rightarrow 0} x \rightarrow 0 \frac{2x}{x}$.
3. $\lim_{x \rightarrow \infty} \sin(x)$.
4. Define $f(x) = \text{sign}(x)$, i.e., $f(x) = +1$ if $x > 0$, $f(x) = -1$ if $x < 0$, and $f(0) = 0$.
6. See answer to problem 4.
8. $f(x) = |x|$.
9. Yes.
10. Yes.
11. Yes.
12. The function must be continuous and the interval finite and closed, e.g., $[a, b]$.
16. $-\frac{1}{3}$
17. 0
18. ∞
19. 0
20. $24x^2 - 2$
21. $\frac{1}{3}x^{-2/3}$
22. $8 \sin^7(x) \cos(x)$
23. $\frac{\sec^2(x)(x^3+1) - \tan(x)(3x^2)}{(x^3+1)^2}$
24. $3 \tan(3x) \sec(3x) \cos(x) + \sec(3x)(-\sin(x))$
25. $\frac{\cos(t)}{2t}$
26. $-\frac{16}{13}$
27. $1\frac{5}{8}$