

1. Find the limits:

- a. $\lim_{x \rightarrow 1} \frac{\ln(x)}{x-1}$ b. $\lim_{\Delta x \rightarrow 0} \ln \frac{(3+\Delta x)^2-9}{\Delta x}$ c. $\lim_{x \rightarrow 0} (1+x)^{1/x}$
d. $\lim_{x \rightarrow -\infty} \frac{x^2+1}{e^x}$ e. $\lim_{x \rightarrow 0} (\sec(x))^{\frac{1}{x^2}}$ f. $\lim_{x \rightarrow 0} \frac{-\ln(\cos(x))}{x^2}$
g. $\lim_{y \rightarrow 0} \frac{\sec(y)-1}{y}$.

2. Find the integrals:

- a. $\int \frac{\ln(x^2)}{x} dx$ b. $\int \frac{e^x}{1+e^x} dx$
c. $\int \frac{x}{1+x^4} dx$ d. $\int_0^3 e^{-\frac{x}{3}} dx$ e. $\int \frac{dx}{9+4x^2}$

3. If $f(x) = \ln(x^2 e^x)$, then what does $f'(2)$ equal?

4. Find $\frac{d}{dx} \log_x(x+1)$

5. Two buildings, one 1 unit high, the other 2 units high are 3 units apart. A solar collector is to be placed between them on the ground, maximizing its angle of exposure. How far should it be from the shorter building?

6. A bicycle is being peddled at the rate of one complete turn of the peddles per second. The peddle sprocket has radius 4 inches and the wheel sprocket has radius 1 inch. The radius of the wheel is 18 inches. The length of the chain is 6 feet and the bicycle is red. How fast is the bicycle moving? If the bicycle is climbing a hill which is 15 degrees from the horizontal and moving 25 miles per hour, how fast is the altitude changing?

7. Derive a rule for finding $\frac{d}{dx}(u^v)$ in terms of $u, v, \frac{du}{dx}$, and $\frac{dv}{dx}$; where u and v are differentiable functions of x . You may use any rules for differentiation you know.

8. Find the value of c such that the line $y = c$ bisects the region bounded by the curves $y = x^2$ and $y = 1$.

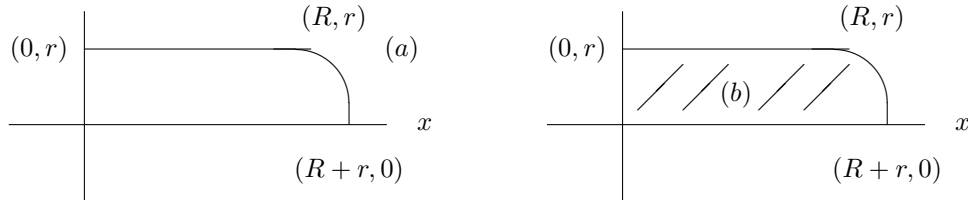
9. The base of a solid is the triangle in the x,y -plane with vertices at $(0,0)$, $(0,1)$, and $(1,0)$. The cross sections perpendicular to the x -axis are squares with one side on the base. Find the volume of the solid.

10. What is the surface area of the solid obtained by revolving the curve $y = x^3$ with $0 \leq x \leq 1$ about the x -axis?

11. What is the center of gravity of a wire joining $(0,0)$ and $(1,0)$ with density $\rho(x) = 1 - x$?

12. Find the surface area $S(R,r)$ of the Diskus of Frisbius. First described by Archimedes in 275 BC, it is obtained by revolving the curve (a) consisting of the line segment joining $(0,r)$ and (R,r) , and the quarter circle of radius r centered

at $(R, 0)$. Let $V(R, r)$ be the volume obtained by revolving the area (b) around the y -axis. Show how to obtain $V(R, r)$ by using $\bar{y}(R, r)$. Explain your formula.



13. Byzantium decays at a rate proportional to the amount present. The half-life is 500 years. What fraction of the original amount is present after 250 years?

Final Exam from Dec 86

1. Prove directly without using any differentiation rules that $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$.
2. True or false.
 - a. A continuous function on an interval is always differentiable except at possibly finitely many points.
 - b. If a function $f(x)$ is differentiable at a point c then it is continuous at c .
 - c. Between any two zeros of a differentiable function lies a zero of its derivative.
 - d. If an interval contains three distinct zeros of a twice differentiable function then it also contains a zero of the function's second derivative.
 - e. If the derivative of a function is always positive then the function cannot have an inflection point.
 - f. The product of two differentiable functions is continuous.
 - g. A continuous function f on an interval $[a, b]$ must achieve its extreme values (minimum and maximum).
 - h. If $f(c)$ is the maximum value of a differentiable function f , then $f'(c) = 0$.
 - i. Suppose $f(x)$ is a continuous function on $[a, b]$ such that for every x in $[a, b]$, $f(x)$ is in $[a, b]$. Then for some c in $[a, b]$, $f(c) = c$.
 From now on assume f , g , and f_k are continuous functions on the interval $[a, b]$.
 - k. $\int_a^b f(x)g(x)dx = \int_a^b f(x)dx \int_a^b g(x)dx$
 - j. $\int_a^b [c_1f(x) + c_2g(x)]dx = c_1 \int_a^b f(x)dx + c_2 \int_a^b g(x)dx$ where c_1, c_2 are constants.
 - l. $\int_a^b [\sum_{k=1}^n c_k f_k(x)]dx = \sum_{k=1}^n [c_k \int_a^b f_k(x)dx]$ where $c_1, \dots, c_n$ are constants.

- m. $\int_a^b [f(x)]^n dx = [\int_a^b f(x) dx]^n$
 n. $\int_a^b \sqrt{f(x)} dx = \sqrt{\int_a^b f(x) dx}$
 o. if $a < c < b$, then $\int_a^c f(x) dx = \int_a^b f(x) dx - \int_c^b f(x) dx$
 p. if $a < c < b$, then $\int_a^c f(x) dx \leq \int_a^b f(x) dx$
 q. Suppose for every x in $[a, b]$, $f(x) \leq g(x)$, then $\int_a^b f(x) dx \leq \int_a^b g(x) dx$
 r. Suppose for every x in $[a, b]$, $f(x) \leq g(x)$, then for every x in $[a, b]$, $f'(x) \leq g'(x)$.

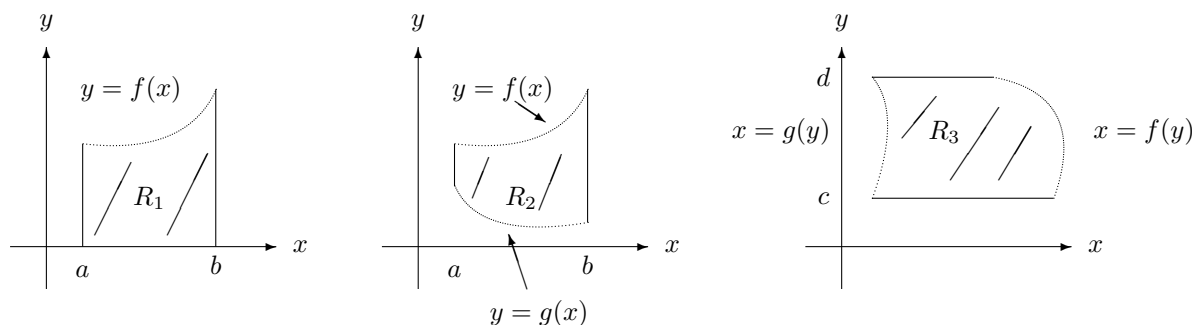
3. Multiple choice:

1. Suppose $\int_1^4 f(t) dt = 6$ and $\int_3^4 f(t) dt = -2$. What is $\int_1^3 (2f(t) + 1) dt =$?
 a. cannot be determined b. 13 c. 18 d. 9
 2. What is $\frac{d}{dx} \int_x^x \sin(t^2) dt =$ a. $\sin(x^4) - \sin(x^2)$ b. $(2x - 1) \sin(x^2)$ c. $2x^2 \sin(x^4) - 2x \sin(x^2)$ d. $2x^2 \cos(x^4) - 2x \cos(x^2)$ e. none of these
 3. $f(x) = \int_1^x \sqrt{1+t^2} dt$, $g(x) = x^2$, $h(x) = f(g(x))$, $h'(1) =$ a. $\sqrt{2}$ b. $2\sqrt{2}$ c. 2 d. 1 e. none of these.
 4. $\lim_{y \rightarrow -\infty} \frac{y-1}{\sqrt{y^2-1}} =$ a. doesn't exist b. 1 c. -1 d. ∞ e. 0

4. Find the following integrals:

- a. $\int \cos(4 - 2x) dx$
 b. $\int \frac{1}{x^2 \sqrt{1 + \frac{1}{x}}} dx$
 c. $\int \tan(\theta) d\theta$
 d. $\int \frac{y^3}{(y^2+1)^3} dy$
 e. (Note definite) $\int_0^1 \frac{x}{x^2+1} dx$

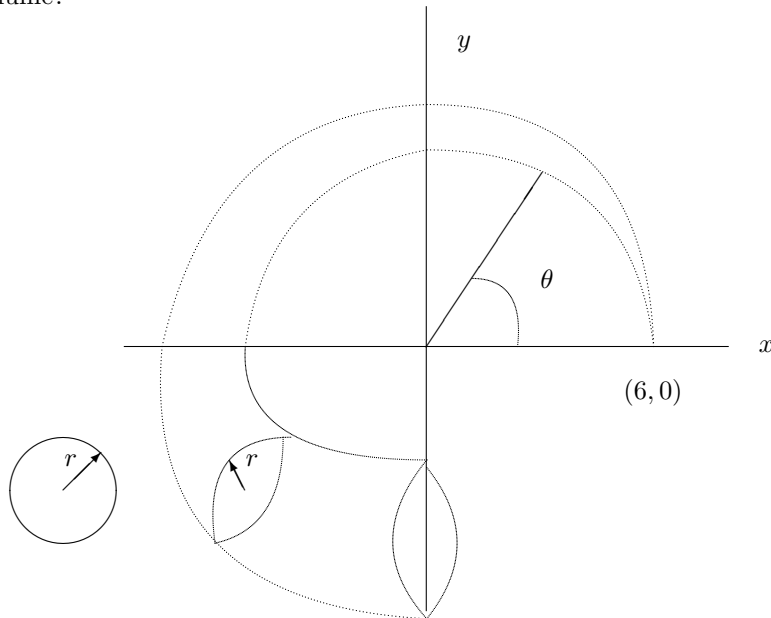
5. Let R_1, R_2 , and R_3 be the regions indicated below.



Express the following as definite integrals.

- (a) The volume of the solid generated when R_1 is revolved about the x-axis.
 (b) The volume of the solid generated when R_2 is revolved about the y-axis.
 (c) The volume of the solid generated when R_3 is revolved about the x-axis.
 (d) The volume of the solid generated when R_3 is revolved about the y-axis.

6. A company wants to manufacture a right circular cylindrical can. The material for the round ends costs 20 cents per sq cm. and the material for the side costs 10 cents per sq cm. The volume is required to be 108π cubic cm. What are the dimensions of the can which is cheapest to manufacture (Assume no waste in cutting the material).
7. The base of a solid is the triangle in the x,y-plane with vertices at (0,0), (0,1), and (1,0). The cross sections perpendicular to the x-axis are squares with one side on the base. Find the volume of the solid.
8. Cornucopia: Horn of Plenty. The center of horn is a circle of radius 6. The cross section at the angle θ of the Horn is a circle of radius $r = \theta$. What is the volume?



Final Exam from May 89

1. A search light is revolving around once every minute, shining on a wall. When the light beam is making a 90 degree angle with the wall you see that it is traveling at the rate of 10 feet per second. How far is the search light from the wall?
2. Use Newton's method to solve $x^3 = x + 1$ with a starting value of $x_0 = 1$. Find x_1 .

3. Use the trapezoidal rule with $n = 4$ to estimate $\int_0^2 \sqrt{1+x^2} \, dx$.
4. Radioactive material decays at a rate proportional to the amount present. Ten megatons of radioactive waste with a half-life of one hundred centuries is buried in the Arizona desert. How much remains after one century?
5. Find the integrals a. $\int \frac{dx}{\sqrt{4-x^2}}$. b. $\int \frac{dx}{x \ln(x)}$.
6. Find the derivative of $10^{\log_2(x)}$.
7. Find the limit $\lim_{x \rightarrow 0} (1+2x)^{1/x}$.
8. A thin plate of constant density is made in a shape which is bounded by the curves $y = x$ and $y = x^3$ and having $0 \leq x \leq 1$. Find the center of mass.
9. A solid is obtained by revolving about the x -axis the area bounded by the curve $y = 4 - x^2$ and the x -axis. Express the volume and surface area as definite integrals, but do **not** solve them.
10. The base of a solid is the triangle in the x,y -plane with vertices at $(0,0)$, $(0,1)$, and $(1,0)$. The cross sections perpendicular to the x -axis are squares with one side on the base. Find the volume of the solid.

Answers

1. a. 1 b. $\ln(6)$ c. e d. ∞ e. $e^{1/2}$ f. $1/2$ g. 0
2. a. $(\ln(x))^2 + C$ b. $\ln(1 + e^x) + C$ c. $1/2 \arctan(x^2) + C$ d. $3(1 - e^{-1})$ e. $1/6 \arctan(\frac{2}{3}x) + C$
3. 2
4. $\frac{\ln(x)/(x+1) - \ln(x+1)/x}{(\ln(x))^2}$
5. 1.472 units
6. 25.70 miles per hour. 6.47 miles per hour.
7. $\frac{d}{dx}(u^v) = u^v(\frac{dv}{dx} \ln(u) + \frac{v}{u} \frac{du}{dx})$
8. $c = (\frac{1}{2})^{2/3}$
9. $1/3$
10. $(\pi/27)(10^{3/2} - 1)$
11. $1/3$
12. $S(R, r) = \pi^2 Rr + 2\pi r^2 + \pi R^2$. $V(R, r) = \int_0^r S(R, t) dt$ so $V(R, r) = \pi^2 Rr^2/2 + 2\pi r^3/3 + \pi R^2 r$.
13. .707107

Answers to Final Exam from Dec 86

2. a. F b. T c. T d. T e. F f. T g. T h. T i. T k. F j. T l. T m. F n. F o. T p. F q. T r. F
3. 1. c. 18 2. e none of these 3. b. $2\sqrt{2}$ 4. c. -1
4. a. $-1/2 \sin(4 - 2x) + C$ b. $-2\sqrt{1 + 1/x} + C$ c. $-\ln|\cos\theta| + C$ d. $1/2(-1/(y^2 + 1) + 1/(2(y^2 + 1)^2) + C$ e. $1/2 \ln(2)$
5. (a) $\int_a^b \pi(f(x))^2 dx$ (b) $\int_a^b 2\pi x(f(x) - g(x)) dx$ (c) $\int_c^d 2\pi y(f(y) - g(y)) dy$ (d) $\int_c^d \pi(f(y)^2 - g(y)^2) dy$
6. radius= 3, height= 12
7. $1/3$
8. $(27/4)\pi^3$

Answers to Final Exam from May 89

1. $\frac{600}{2\pi}$ feet
2. 1.5
3. 2.97653
4. 9.93092 megatons
5. a. $\ln(|\ln|x||) + C$ b. $\arcsin(x/2) + C$
6. $10^{\log_2(x)} \frac{\ln(10)}{x \ln(2)}$
7. e^2
8. $(\frac{8}{15}, \frac{8}{21})$
9. Volume= $\int_{-2}^2 \pi(4 - x^2)^2 dx$ Surface area= $\int_{-2}^2 2\pi(4 - x^2)\sqrt{1 + 4x^2} dx$
10. $1/3$