

1. $\lim_{x \rightarrow \infty} \frac{1+x-2x^2}{3x^2-4x+1}$

2. $\lim_{x \rightarrow 0} \frac{-3 \sin^2(2x)}{-x^2}$

derivatives

3. Find y'' if

$$y = \frac{5}{12}x^4 - \frac{7}{12}x^3 + \frac{1}{2}x^2 - 2x + 5$$

4. Show that if $f(x) = \sqrt{x}$ then $f'(x) = \frac{1}{2\sqrt{x}}$ directly without using any rule for taking derivatives.

5. State the quotient rule for finding the derivative of u/v where u and v are functions of x . Give a proof of this formula directly from the definition of derivative and not using any other rule for taking derivatives.

6. Find $\frac{dy}{dx}$ if $y = (6 - \sqrt{x})^{7/4}$.

7. Find $\frac{dy}{dx}$ if $y = \tan(\sin^2(x))$.

8. Assume that y is a differentiable function of x . Use implicit differentiation to find $\frac{dy}{dx}$ if $6x^3 + 8 \cos(xy) = 2$.

9. The following parametric equations give the position of a particle moving in the plane.

$$y = t^2, \quad x = t^3 - t$$

Find $\frac{d^2y}{dx^2}$ at the point the particle is at when $t = 1$.

tangent lines and linear approximation

10. Find the equation of the tangent line to the curve $x^2 + 4xy - y^2 = 11$ at the point $(2, 1)$.

11. Write an equation for the line tangent to $y = x^3 + 2x^2 - 2$ at the point $(2, 14)$.

12. Let $f(x) = -x^3 - 3x + 2$. Find the linearization of f at $a = 1$ use it to approximate $f(1.01)$.

13. The curve $y = ax^2 + bx + c$ passes through the point $(1, 4)$ and is tangent to the line $y = 2x$ at the origin. Find a, b , and c .

Newton's method

14. Use Newton's method to find x_2 if $f(x) = x^3 + 3x^2 + 4x + 1$ where $x_0 = -1$.

True and False

15. (True or False) If f is a function which is continuous at a a then f is differentiable at a .

16. (True or False) If f is a function which is differentiable at every real x and $f(a) < 0$ and $f(b) > 0$, then there exist a c between a and b such that $f(c) = 0$.

misc

17. Define $f(-4)$ in a way that extends

$$f(x) = \frac{x^2 - 16}{x^2 - x - 20}$$

to be continuous at $x = -4$.

18. Prove that (a) $\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$ or assume (a) and prove that (b) $\frac{d}{dx} \sin(x) = \cos(x)$.

Exam from Feb 89

1. A particle is moving along a straight line at constant acceleration. At time $t = 0$ it has moved 0 units, at time $t = 1$ it has moved a total of 3 units, and at time $t = 2$ it has moved a total of 12 units. At time $t = 3$ what is its velocity?

2. For each of these say whether it is true or false.

a. A continuous function on an interval is always differentiable except for at most finitely many points.

b. If a function $f(x)$ is differentiable at c then it is continuous at c .

c. If $f(x) = (x^3 + x + 1)$ then there must be a real number c with $-1 < c < 1$ such that $f(c) = 0$.

d. The product of two differentiable functions is continuous.

3. Calculate the following limits or show they don't exist:

a. $\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$

b. $\lim_{x \rightarrow 1} \frac{x-1}{x^2-1}$

c. $\lim_{x \rightarrow 0} x^2 \csc^2(3x)$

d. $\lim_{x \rightarrow 0} \sin\left(\frac{1}{x}\right)$

e. $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+1}}{\sqrt{2x^2+x+1}}$

4. a. Suppose $y = 2x^2 - 3x + 7$. What is $\frac{dy}{dx}$?

b. Suppose $p = \sin(y) \sec(y)$. What is $\frac{dp}{dy}$?

c. Suppose $r = \frac{\sin(2\theta)}{\tan(\theta)}$. What is $\frac{dr}{d\theta}$?

d. Suppose $y = \sin(x^2 + x - 1)$. What is $\frac{dy}{dx}$?

e. What is $\frac{d}{dx}((x^2 + 1)^{5c})$ where c is a constant?

5. In the limit $\lim_{x \rightarrow 2} x^2 = 4$ find a $\delta > 0$ such that for any x , if $|x - 2| < \delta$ then $|x^2 - 4| < 0.01$.
6. Use Newton's method to find approximate solutions to the equation $x^2 = 2$. Use the starting value $x_0 = 2$ and find x_1 and x_2 .
7. Use linear approximation to find an approximation to $(99)^{3/2}$.
8. Suppose that the **derivative** of $f(x)$ is $\tan^2(x)$. What is the derivative of $f(\sin(x))$ with respect to x ?
9. Suppose $x = \sin(t)$ and $y = \sin(2t)$. Find the slope of the tangent line to this curve when $t = \pi$.
10. Suppose y is a differentiable function of x satisfying $x^2 + xy^2 + 3y = 12$ for every x . What is $\frac{dy}{dx}$?
11. Without using any rules of differentiation show directly from the definition of the derivative that
- $$\frac{d(\frac{1}{x})}{dx} = -\frac{1}{x^2}$$
12. State the product rule for differentiating uv where u and v are differentiable functions of x . Derive this formula directly from the definition of the derivatives of u and v .

Answers

1. $-2/3$
2. 12
3. $y'' = 5x^2 - \frac{7}{2}x + 1$
6. $\frac{dy}{dx} = \frac{-7(6-\sqrt{x})^{3/4}}{8\sqrt{x}}$
7. $\frac{dy}{dx} = \sec^2(\sin^2(x))2\sin(x)\cos(x)$
8. $\frac{dy}{dx} = \frac{18x^2 - 8y \sin(xy)}{8x \sin(xy)}$
9. -1
10. $y - 1 = (-4/3)(x - 2)$
11. $-20x + y = -26$
12. $L(x) = -6x + 4$ and $f(1.01)$ is approximately $L(1.01) = -2.06$
13. $y = 2x^2 + 2x$
14. $-1/4$
15. False. The function $f(x) = |x|$ is continuous at 0 but not differentiable at 0.
16. True. Differentiable implies continuous and continuous function have the intermediate value property.
17. Define $f(-4) = 8/9$

Answers to Exam from Feb 89

1. 18
2. a. False. Consider a saw tooth curve where the teeth get smaller and smaller.
- b. True. If the limit

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$
 exists then it must be that $\lim_{x \rightarrow a} f(x) = f(a)$.
- c. True f is a differentiable function hence it is continuous and so the intermediate value theorem applies.
- d. True. The product of differentiable functions is differentiable and all differentiable functions are continuous.
3. a. 1 b. $\frac{1}{2}$ c. $\frac{1}{9}$ d. does not exist e. $\sqrt{\frac{1}{2}}$
4. a. $4x - 3$ b. $\cos(y)\sec(y) + \sin(y)\tan(y)\sec(y)$ c. $\frac{2\cos(2\theta)\tan(\theta) - \sin(2\theta)\sec^2(\theta)}{\tan^2(\theta)}$
- d. $(\cos(x^2 + x - 1))(2x + 1)$ e. $5c(x^2 + 1)^{5c-1}(2x)$
5. $\delta = .002$
6. $x_1 = 1.5, x_2 = 1.4166666$
7. $985 = 1000 - 15(99 - 100)$
8. $(\tan^2(\sin(x)))\cos(x)$
9. -2
10. $\frac{dy}{dx} = \frac{-2x - y^2}{2xy + 3}$