

Show all work. Circle your answers. Use back of sheets if necessary. Do not simplify.

Hand in to your TA. Circle your TA's name:

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Name _____

Score

1. _____ (25)

2. _____ (46)

3. _____ (30)

4. _____ (24)

5. _____ (25)

6. _____ (25)

7. _____ (25)

Total _____ (200) Grade _____

1. Prove directly without using any differentiation rules that

$$\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$$

2. True or false. Fill in T or F in the blank provided.

_____ 2.1. A continuous function on an interval is always differentiable except at possibly finitely many points.

_____ 2.2. If a function $f(x)$ is differentiable at a point c then it is continuous at c .

_____ 2.3. Between any two zeros of a differentiable function lies a zero of its derivative.

_____ 2.4. If an interval contains three distinct zeros of a twice differentiable function then it also contains a zero of the function's second derivative.

_____ 2.5. If the derivative of a function is always positive then the function cannot have an inflection point.

_____ 2.6. The product of two differentiable functions is continuous.

_____ 2.7. A continuous function f on an interval $[a,b]$ must achieve its extreme values (minimum and maximum).

_____ 2.8. If $f(c)$ is the maximum value of a differentiable function f , then $f'(c) = 0$.

_____ 2.9. Suppose $f(x)$ is a continuous function on $[a,b]$ such that for every x in $[a,b]$, $f(x)$ is in $[a,b]$. Then for some c in $[a,b]$, $f(c)=c$.

From now on assume f , g , and f_k are continuous functions on the interval $[a,b]$.

_____ 2.10. (Kerr) $\int_a^b f(x)g(x)dx = \int_a^b f(x)dx \int_a^b g(x)dx$

_____ 2.11. (Kerr) $\int_a^b [c_1 f(x) + c_2 g(x)]dx = c_1 \int_a^b f(x)dx + c_2 \int_a^b g(x)dx$ where c_1, c_2 are constants.

_____ 2.12. (Kerr) $\int_a^b [\sum_{k=1}^n c_k f_k(x)]dx = \sum_{k=1}^n [c_k \int_a^b f_k(x)dx]$ where $c_1, \dots, c_n$ are constants.

_____ 2.13. (Kerr) $\int_a^b [f(x)]^n dx = [\int_a^b f(x)dx]^n$

_____ 2.14. (Kerr) $\int_a^b \sqrt{f(x)}dx = \sqrt{\int_a^b f(x)dx}$

_____ 2.15. (Kerr) if $a < c < b$, then $\int_a^c f(x)dx = \int_a^b f(x)dx - \int_c^b f(x)dx$

_____ 2.16. (Kerr) if $a < c < b$, then $\int_a^c f(x)dx \leq \int_a^b f(x)dx$

_____ 2.17. Suppose for every x in $[a,b]$, $f(x) \leq g(x)$, then $\int_a^b f(x)dx \leq \int_a^b g(x)dx$

_____ 2.18. Suppose for every x in $[a,b]$, $f(x) \leq g(x)$, then for every x in $[a,b]$, $f'(x) \leq g'(x)$.

The following problems are multiple choice. Fill in the correct answer in the blank provided.

_____2.19. Suppose $\int_1^4 f(t)dt = 6$ and $\int_3^4 f(t)dt = -2$. What is $\int_1^3 (2f(t) + 1)dt = ?$

a. cannot be determined b. 13 c. 18 d. 9 e. none of these

_____2.20. What is $\frac{d}{dx} \int_x^{x^2} \sin(t^2) dt =$

a. $\sin(x^4) - \sin(x^2)$

b. $(2x - 1) \sin(x^2)$

c. $2x^2 \sin(x^4) - 2x \sin(x^2)$

d. $2x^2 \cos(x^4) - 2x \cos(x^2)$

e. none of these

_____2.21. (Kang) $f(x) = \frac{1}{x^2+1}$. Graph of f is

_____2.22. (Kang) $f(x) = \int_1^x \sqrt{1+t^2}dt$, $g(x) = x^2$, $h(x) = f(g(x))$, $h'(1) =$

a. $\sqrt{2}$ b. $2\sqrt{2}$ c. 2 d. 1 e. none of these.

_____2.23. (Kang) $\lim_{y \rightarrow -\infty} \frac{y-1}{\sqrt{y^2-1}} =$

a. doesn't exist b. 1 c. -1 d. ∞ e. 0

3. Find the following integrals (all from 4.4):

$$\int \cos(4 - 2x) \, dx$$

$$\int \frac{1}{x^2 \sqrt{1 + \frac{1}{x}}} dx$$

$$\int \tan(\theta) \, d\theta$$

$$\int \frac{y^3}{(y^2 + 1)^3} dy$$

$$(\text{ Note definite }) \int_0^1 \frac{x}{x^2 + 1} dx$$

4. (Kerr) Let R_1, R_2, R_3 , and R_4 be the regions indicated below:

Express the following as definite integrals.

(a) The volume of the solid generated when R_1 is revolved about the x-axis.

(b) The volume of the solid generated when R_2 is revolved about the y-axis.

(c) The volume of the solid generated when R_3 is revolved about the x-axis.

(d) The volume of the solid generated when R_4 is revolved about the y-axis.

5. (Teske) A company wants to manufacture a right circular cylindrical can. The material for the round ends costs 20 cents per sq cm. and the material for the side costs 10 cents per sq cm. The volume is required to be 108π cubic cm. What are the dimensions of the can which is cheapest to manufacture (Assume no waste in cutting the material).

6. (Teske) (prob 1 in section 6.1) The base of a solid is the triangle in the x,y-plane with vertices at (0,0), (0,1), and (1,0). The cross sections perpendicular to the x-axis are squares with one side on the base. Find the volume of the solid.

7. Cornucopia: Horn of Plenty. The dotted line is a circle of radius 6. The cross section at the angle θ of the Horn is a circle of radius $r = \theta$. What is the volume?