

Show all work. Use back of sheets if necessary. Do not simplify.

Hand in to your TA. Circle your TA's name:

Alexander Kang Kerr Linton Morrison Teske

Name _____

Score

1. _____ (12)

2. _____ (16)

3. _____ (15)

4. _____ (15)

5. _____ (12)

6. _____ (15)

7. _____ (15)

Total _____ (100) Grade _____

1. Find the area of the region under the curve $y = \cos(x)$ and between the lines $x = 0$ and $x = \pi/2$.

2. For each of these say whether it is true or false and give a one sentence explanation:

a. A continuous function on an interval is always differentiable except for at most finitely many points.

b. If a function $f(x)$ is differentiable at a point c then it is continuous at c .

c. Suppose $g'(x) = 0$ for all x , then g is a constant function.

d. Between any two zeros of a differentiable function lies a zero of its derivative.

e. If the derivative of a function is always positive then the function cannot have an inflection point.

f. If $f(x) = (x^3 + x + 1)e^x$ then there must be a real number c with $0 < c < 1$ such that $f'(c) = 3e - 1$.

g. If $f(x) = (x^3 + x + 1)e^x$ then there must be a real number c with $-1 < c < 1$ such that $f(c) = 0$.

h. Suppose $g'(x) > 0$ on an interval (a,b) then $g(x)$ is a decreasing function.

3. Find the following limits:

a. $\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$

b. $\lim_{x \rightarrow 1} \frac{\log(x)}{x-1}$

c. $\lim_{x \rightarrow 0} (1+x)^{1/x}$

d. $\lim_{x \rightarrow 1} \frac{x-1}{x^2-1}$

e. $\lim_{\Delta x \rightarrow 1} \log \frac{(3+\Delta x)^2-9}{\Delta x}$

4. A man has 16 miles of fencing for inclosing two separate lots, one of which is to be a square and the other a rectangle which is three times as long as it is wide. Find the dimensions of each lot so that the total area which is inclosed shall be a minimum.

5. a. The function $f(x)$ on the interval $[1,8]$ has the following graph. Which points are

points where f is not continuous

points where f is continuous but not differentiable

points where f is differentiable but not continuous

b. The function $g(x)$ on the interval $(1,8)$ has both a first and second derivative at every point. Find the intervals where

$g'(x)$ is positive

$g'(x)$ is negative

$g''(x)$ is positive

$g''(x)$ is negative

6. A piston is being driven back and forth by a crankshaft which is turning (see diagram). The points A,B,C are swivel joints. The point B which is the axis of the crankshaft is held fixed while the point A rotates around counterclockwise exactly once per unit of time. The point C on the piston moves horizontally back and forth. The two connecting arms AB and AC are rigid and of length 1 unit and 3 units respectively. Find the speed the piston (point C) is moving when the point A is directly above the point B. What is the speed when the angle ABC is 45 degrees?

7. Define $\int_a^b f(x) dx$. Prove the first part of the
FUNDAMENTAL THEOREM OF CALCULUS:

Suppose that $f(t)$ is a continuous function on the interval $[a,b]$ and we
define

$$F(x) = \int_a^x f(t) dt$$

then $F'(x) = f(x)$.