

5 points per problem.
Show all work and explain your answer.

1. (a) Draw a picture of the region R of the plane which is under the curve $f(x) = x^2$ and above the interval $[0, 1]$ on the x -axis. Shade the region R .

(b) Let R_n be sum of the areas of the following n rectangles which approximate the area of R . Use rectangles of equal width and use the right hand endpoint of each interval and f to find their height. Draw a picture of these rectangles for $n = 3$ and calculate R_3 .

(c) Find a summation (Σ) expression for R_n .

(d) Simplify the expression for R_n .

(e) Find $\lim_{n \rightarrow \infty} R_n$.

2. (a) $\int_1^3 (3 - 2f(x))dx = 9$.

What is $\int_1^3 f(x)dx$?

(b) $\int_{-1}^1 g(x)dx = 3$, $\int_{-2}^2 g(x)dx = 9$, and $\int_{-2}^{-1} g(x)dx = 2$.

What is $\int_1^2 g(x)dx$?

3. Let $h(x) = \int_{x^3}^0 e^{t^2} dt$. Find $h'(x)$.

4. (a) Find $\int \sin(2x + 1)dx$.

(b) Find $\int_0^1 (x^2 + 1)dx$. (This is a definite integral.)

(c) Find $\int \frac{\arcsin(x)}{\sqrt{1-x^2}} dx$.

The final exam will be
Monday December 15 at 5:05pm
in 494 Van Hise.

Answers

1. (b) $\frac{14}{27}$ (c) $\sum_{k=1}^n \left(\frac{k}{n}\right)^2 \left(\frac{1}{n}\right)$ (d) $\frac{(n+1)(2n+1)}{6n^2}$ (e) $\frac{1}{3}$
2. (a) $-\frac{3}{2}$ (b) 4
3. $-3x^2 e^{x^6}$
4. (a) $-\frac{1}{2} \cos(2x + 1) + C$ (b) $\frac{4}{3}$ (c) $\frac{(\arcsin(x))^2}{2} + C$