Exam 4 A. Miller Spring 97 Math 221

5 points per problem.
Show all work and explain your answer.

1. Find all critical points of the function $F(x) = x^{\frac{4}{5}}(x-4)^2$.

2. (a) Verify that $f(x) = 2x^3 + x^2 - x - 1$ satisfies the hypotheses of the Mean Value Theorem on the interval $[0, 2]$.

(b) Find all numbers c that satisfy the conclusion of the Mean Value Theorem.

3. Find all intervals where $f(x) = x^4 - 8x^2 + 1$ is

(a) increasing

(b) decreasing

(c) concave up

(d) concave down

4. Graph $f(x) = \frac{\ln(x)}{x}$. Identify the domain, intercepts, asymptotes, symmetries, increasing or decreasing intervals, local max or min, concave up and down intervals, and inflection points.

5. A Norman window has the shape of a rectangle surmounted by a a semicircle. (Thus the diameter of the semicircle is equal to the width of the rectangle.) If the perimeter of the window is 30 find the dimensions of the window so that the greatest possible amount of light is admitted.

6. Find the most general antiderivative of

$$f(x) = \frac{x^2+1}{x} + 3\sqrt{x} - 2(1+x^2)^{-1}$$

Answers

1. (4.1-35) $0, 4, \frac{8}{7}$

2. (4.2-12) (a) Any polynomial is differentiable and continuous everywhere. (b) $\frac{f(b)-f(a)}{b-a} = 9$, $c = \frac{-1+\sqrt{61}}{6}$ is the only solution of $f'(c) = 9$ in the interval $[0, 2]$.

3.

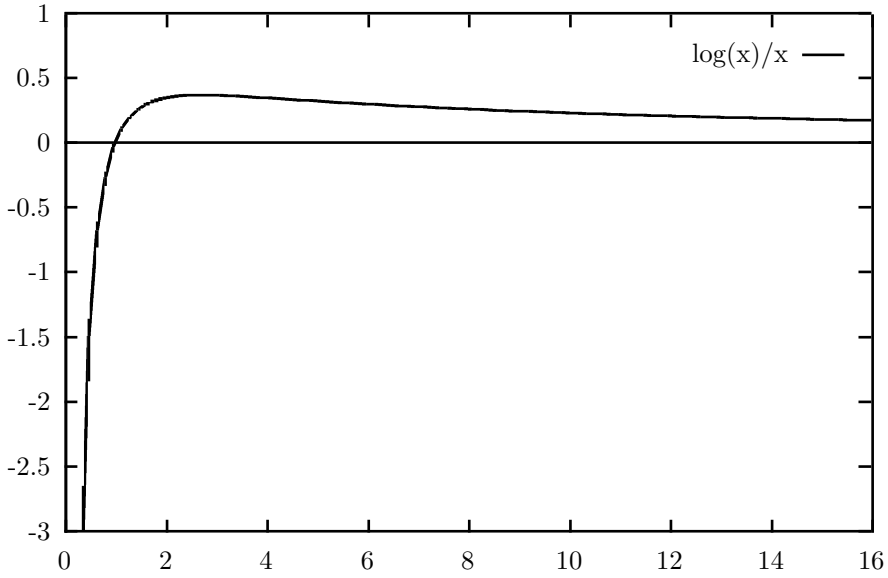
(a) $(-2, 0), (2, \infty)$

(b) $(-\infty, -2), (0, 2)$

(c) $(-\infty, -\sqrt{\frac{4}{3}})$ $(\sqrt{\frac{4}{3}}, \infty)$

(d) $(\sqrt{\frac{4}{3}}, -\sqrt{\frac{4}{3}})$

4. (4.5-48) Domain is the set of positive reals. Intercept at $x = 1$. Maximum at $x = e$. Inflection point at $x = e^{\frac{3}{2}}$.



5. (4.7-26) The width is $\frac{60}{\pi+4}$.

6. $\frac{x^2}{2} + \ln|x| + 2x^{\frac{3}{2}} - 2\arctan(x) + C$.