Exam 2 A. Miller Spring 97 Math 221

Show all work. 5 points per problem.

1. Find the derivative of $g(x) = \frac{1}{\sqrt{x}}$ directly using the definition of derivative. (Do not use the rules of differentiation in this problem, except to check your answer.)

2. Find the derivatives. (From here on you may use the rules of differentiation.)

(a) $h(t) = (t^2 + 1)\sin(t)$

(b) $k(x) = \frac{x^3-1}{x^2+1}$

(c) $l(x) = x - x^{-1} + 3x^{-4}$

(d) $g(t) = t^{\sqrt{2}}$

(e) $f(x) = (\sqrt{2})^5$

3. The mass of the part of a metal rod that lies between its left end and a point x feet to right is x^3 pounds.

(a) Find the mass of the metal between its left end and a point 2 feet to the right.

(b) Find the average density of the rod between $x = 2$ and $x = 2.1$ feet.

(c) Find the linear density when $x = 2$.

4. Find the $\lim_{x\to 0} \frac{1-\cos(x)}{3x^2}$.

5. Find the derivatives.

(a) $h(u) = (2 + 3u^2)^{99}$

(b) $k(t) = \cos^2(t)$

(c) $f(x) = (x + \frac{1}{x})^5$

(d) $g(\theta) = \sin(\tan(\theta))$

(e) $h(t) = \left(\frac{1+t^5}{\sec(3t)}\right)^{-3}$

6. Suppose $y^5 + xy = 3$ defines y implicitly as a function of x. Find $\frac{dy}{dx}$ at the point $(2, 1)$.

7. Suppose $f(x) = \frac{1}{1-x}$. What is $f^{(17)}(x)$?

8. Find the linear and quadratic approximation to $f(x) = \sqrt{x}$ at the point $a = 9$ and use them to approximate $f(9.1)$.

Answers

1.

$$\frac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h}=\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x+h}\sqrt{x}}=\frac{(\sqrt{x}-\sqrt{x+h})}{(h\sqrt{x+h}\sqrt{x})}\frac{(\sqrt{x}+\sqrt{x+h})}{(\sqrt{x}+\sqrt{x+h})}=$$

$$\frac{x-(x+h)}{h\sqrt{x+h}\sqrt{x}(\sqrt{x}+\sqrt{x+h})}=\frac{-1}{\sqrt{x+h}\sqrt{x}(\sqrt{x}+\sqrt{x+h})}$$

$$\to\frac{-1}{\sqrt{x}\sqrt{x}(\sqrt{x}+\sqrt{x})}=\frac{-1}{2x\sqrt{x}}\text{ as }h\to 0$$

2. (a) $2t\sin(t)+(t^2+1)\cos(t)$ (b) $\frac{3x^2(x^2+1)-(x^3-1)(2x)}{(x^2+1)^2}$ (c) $1+x^{-2}-12x^{-5}$ (d) $\sqrt{2}t^{\sqrt{2}-1}$ (e) 0

3. (a) 8 (b) $\frac{(2.1)^3-2^3}{.1}$ (c) 12

4. $\frac{1}{6}$

5. (a) $99(2+3u^2)^{98}(6u)$ (b) $-2\cos(t)\sin(t)$ (c) $5(x+\frac{1}{x})^4(1-\frac{1}{x^2})$ (d) $\cos(\tan(\theta))\sec^2(\theta)$ (e) $-3\left(\frac{1+t^5}{\sec(3t)}\right)^{-4}\frac{5t^4\sec(3t)-3(1+t^5)\tan(3t)\sec(3t)}{\sec^2(3t)}$

6. $-\frac{1}{7}$

7. $\frac{17!}{(1-x)^{18}}$

8. $L(x)=\frac{1}{6}(x-9)+3$, $L(9.1)=\frac{1}{60}+3$, $Q(x)=-\frac{1}{216}(x-9)^2+\frac{1}{6}(x-9)+3$, $Q(9.1)=-\frac{1}{21600}+\frac{1}{60}+3$