Exam 1 A. Miller Fall 97 Math 221

Show all work. 5 points per problem.

1. Sketch the graph of the function

$$g(x) = \begin{cases} 1-x & \text{if} \quad x \le -1 \\ x+3 & \text{if} \quad -1 < x < 2 \\ 2x-1 & \text{if} \quad 2 \le x \end{cases}$$

Use the graph to state the value of each of the following limits, if it exists.

(a) $\lim_{x \to -1^-} g(x)$

(b) $\lim_{x \to -1^+} g(x)$

(c) $\lim_{x \to 0} g(x)$

(d) $\lim_{x \to 2^-} g(x)$

(e) $\lim_{x \to 2^+} g(x)$

2. Evaluate the limit, if it exists. $\lim_{x \to 2} \frac{x^4-16}{x-2}$.

3. State the definition of $f(x)$ is continuous at a. Use the definition of continuous and the properties of the limit to show that $f(x) = \frac{1}{x+3}$ is continuous at $x = 1$.

4. Find the horizontal and vertical asymptotes of the curve

$$y = \frac{2x^3 - x^2 + 4x}{27 - x^3}$$

5. Find the equation of the tangent line to the curve

$$y = \frac{x}{2-x}$$

at the point $(0, 0)$.

Answers

1. (a) 2 (b) 2 (c) 3 (d) 5 (e) 3
2. 32
3. f is continuous at a iff $\lim_{x\to a} f(x) = f(a)$.

$$f(1) = \frac{1}{1+3} = \frac{1}{4}$$

$$\lim_{x\to 1} \frac{1}{x+3} = \frac{\lim_{x\to 1} 1}{\lim_{x\to 1} x + \lim_{x\to 1} 3} = \frac{1}{1+3} = \frac{1}{4}$$

4. Verticle asymptote $x = 3$. Horizontal asymptote $y = -2$.
5. $y = \frac{1}{2}x$